REQUIREMENTS ON A MATHEMATICAL NOTATION FOR THE BLIND

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Abstract
The integration of blind people into learning and working environments of the sighted needs among other things a possibility to write and represent mathematical expressions in an appropriate way. There exist different concepts how to form a mathematical notation for the blind. This contribution presents in a first step a discussion of the variety of demands for a mathematical notation. In a second step it is shown that the "Stuttgarter Mathematikschrift für Blinde" (SMSB) meets these requirements. Particularly it is possible to transform SMSB-terms into a graphical representation, which is familiar to the sighted.

1. Introduction

Promoting the integration of blind students of every age into mainstream education and of blind adults in working positions demands for an adequate notation for mathematical facts in tactile form. Besides a pure possibility to express mathematical terms, there exists a variety of additional needs for a suitable mathematical notation. Some of the requirements arise from a co-operated working of blind and sighted persons, others from the aim to reach also blind children. The existing mathematical notations for the blind regard only a subset of the requirements and ignore others.

2. Requirements on a Mathematical Notation for the Blind

The field of mathematics and its language yields an appropriate own mathematical notation. To open this language to blind persons an only arbitrary notation is not sufficient. It necessitates a lot of specific features.

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R1 The mathematical notation must be readable by finger.

The understanding of complex mathematical terms has to be supported by the notation. Where sighted persons can grasp the visual representation of a term, the blind have to acquire the context character by character. A good orientation for a suitable design is the clearly spoken term. A prefix notation is useful. It is not necessary for simple sums or differences like 5x+12y or 6-15x, nor needs a power with a simple exponent like x^2 a prefix.

R2 The number of characters in a mathematical expression should be as low as possible.

The understanding of a complex term increases with the compactness and clarity of the notation. Therefor, there exists a lot of mathematical symbols like + for PLUS, - for MINUS or √ for ROOT. They support the comprehension of complex coherence. In designing a mathematical notation for the blind, the same abstraction should take place. A one to one translation of spoken mathematical symbols into tactile ones should be preferred.

R3 Tactile characters should be understandable intuitively.

The mathematical notation for the sighted includes many visual symbols. For instance, arrows are used in connection with a vector or a limit. Such symbols are used to simplify the understanding of an object and its function or the meaning of a symbol. The same should be claimed for the blind. Especially syntactical symmetries that imply semantic symmetries like ( ), < > or {} should also be symmetric in the notation for the blind.

R4 The notation should be usable in a computer based learning and working environment.

The use of computers made the integration of blind students into mainstream education more feasible, in many cases possible at all. It also opened occupations to blind people together with the sighted. Therefore, it is indispensable that the computer supports the application of a mathematical notation for the blind.

R5 The joint education of blind and sighted has to be supported.

A twofold representation of a mathematical term is necessary. The notation for the blind must be transformable character by character into a notation for the sighted and vice versa. It should also be possible to transform a written term into the typical, often two-dimensional representation for the sighted. If this condition is fulfilled, a teacher has to write work sheets only once to prepare the version for the sighted and the tactile one for the blind.

Some of these requirements are more or less critical dependent on the target group. For blind scientists it is possible to abandon the strong demand of a one to one translation for the characters as well as for the symbols. But if one considers blind children at their first years of school in the mainstream education, it should be evident that it is an excessive demand of the children to learn two mathematical notations, the spoken one and their special one for reading and writing. Blind scientists are usually familiar with the spoken mathematical notation and therefore are able to learn a second and new version.

3. Existing Mathematical Notations

The mathematical notation for the blind, which has the longest tradition, is the notation of Marburg in Germany. It was published for the first time in 1930 [3]. The last edition is from 1992 [2]. Changes concerning technical possibilities and requirements of a modern notation have provoked the development of further and new attributes of a suitable design. Most of the existing notations fulfil only parts of the above requirements R1-R5.

3.1 The Marburg Notation

This 6-dot-notation provides for 64 characters including the blank. This does not even allow representing unequivocally the small and capital letters, the ten digits, punctuation marks and a few special characters. Consequently, in this notation a definite representation of mathematical symbols in Braille and print is impossible. The mathematical notation was developed for the communication between blind persons. Their mechanical machines for writing were prepared for 6-dot-Braille.
A digit i is announced by a number sign and followed by the i-th letter of the alphabet. For instance 7 is written by #g, 4987 by #dihg. A letter within a formula must have a character placed in front, which makes sure that it is no meaningful character in contracted Braille. It consists of dot 6. These features are in contrast to R2 and R5

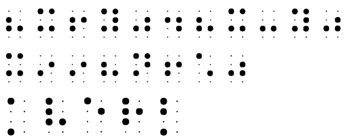
Simple fractions begin with a number sign, which is followed by numerator and denominator, whose dots are lowered in their Braille cell. 4/7 starts with a number sign followed by the forth letter of the alphabet, which is letter d. If the dots of the seventh letter, g, are lowered, the pattern of dots means *equal*. The fraction 4/7 is therefor written as #d=, in visible Braille ⠼⠙⠶

This shows another disadvantage of the Marburg system. The written communication between the blind and the sighted is nearly impossible. There exist several characters, which have no semantic correspondence in print. Therefore the Marburg notation does not meet requirement R4.

3.2 The ASCII-Notation of Karlsruhe

Another notation for the Blind is the ASCII-mathematical notation of the university of Karlsruhe. This notation is platform independent, because it uses exclusively the 128 characters of the 7-bit-ASCII as it has been standardised as DIN 32980 in December 1986. Unfortunately, it is not designed as a prefix notation. The blind have to read many characters before they are able to identify the mathematical term definitely. We show examples in ASCII-notation, as they are typed and shown on the screen, beside it their visible tactile representation.

```
(x+y)÷(x-y)
x**(n+1)
AverB
```



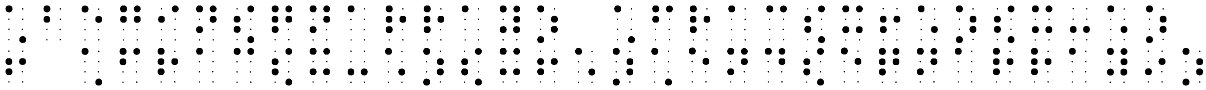
3.3 LaTeX

A widespread mathematical representation is LaTeX. It enables authors to write publications by themselves and produce a version, which is ready for electronic publishing. The text including formulas is typed on a computer keyboard using characters of the 7-bit-ASCII. An advantage, which seems to make LaTeX attractive for schools, is the possibility to adapt the notation to personal preferences as commands in the own language or personal abbreviations. Programs of the LaTeX-packet print the terminating translation of the whole text including the mathematical passages and show it on the screen.

LaTeX has been successful in a scientific environment, especially in universities, where many scientists write their publications by themselves. An advantage of LaTeX is its prefix notation. The main disadvantage for the blind is that symbols are displayed as strings. They contain a command out of several characters. Therefor, the blind have to read more characters than necessary.

Example in LaTeX:

\begin{qnarray} \frac{x+1}{x-1}+\frac{x-1}{x+1}\end{qnarray*}

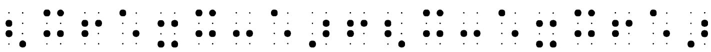


(LaTeX-version in visible Braille if the Braille characters are coded according to the Stuttgart-character-set SZ Braille)

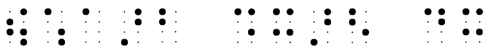
4. The "*S*tuttgarter *M*athematik*s*chrift für *B*linde" (SMSB)

The "Stuttgarter Mathematikschrift" is based on 8-dot-Braille from its beginning in 1980 [4] and is used on computer. It is used analogue to the notation of the sighted and follows many features of the Marburg notation. Since 1999, SMSB-terms can be transformed into a graphical representation. The number of characters in a mathematical expression in SMSB equals the representation in the corresponding version for sighted readers (see R1and R2).

▌x+1▬x-1▌ + ▌x-1▬x+1▌ (1) SMSB-version *SZ Schwarzschrift)*

 (2) SMSB-version type *SZ Braille*)

The integral from a to b of $3x^2$ is written as

∫↓a↑b 3x↑2 dx (1)  (2)

After marking the SMSB-terms (1) and converting them by our transformation-macro in typing the keys Strg and u ("umwandeln") at a time, they look like

$$\frac{x+1}{x-1} + \frac{x-1}{x+1} \quad \text{and} \quad \int_a^b 3x^2 \;dx \qquad (3)$$

A teacher can write a worksheet for his class in SMSB in using the type *SZ Schwarzschrift*. This source (1) can be copied and represented in visible Braille (2) by changing the type into *SZ Braille. S*ighted persons can compare the output for the blind on the Braille-output-device with the printed materials. In a further copy the mathematical terms are transformed as described into their two-dimensional representation (3)

4.1 Fulfilment of the Requirements for a Mathematical Notation for the Blind

R1: SMSB is readable by finger, each character has a definite representation as a printed symbol.

R2: Mathematical expressions are short. SMSB provides tactile characters for symbols like

< > ⊂ ⊃ ∈ → ← √ ∫

We announce only letters of foreign alphabets like Greek letters or letters of the old German alphabet, which are used for instance for angles and for vectors respectively. We do not need an announcement for trigonometric functions, as it is necessary in the Marburg notation.. We use the same letters as the sighted. These are sin, cos, tanh, etc.

R3: SMSB uses characters, which can be understood intuitively by a person, who is familiar with mathematics. Symmetric Braille symbols are symmetric in meaning:

( ) [ ] { } 〈 〉

↓ ↑ ↘ ↗ a ↓ i x ↑ 5 x ↗ 2 n + 5 ♥

/ \ – _____ | used for instance in names of sets: |N

R4: SMSB is usable in a computer based learning and working environment.

All symbols, which are needed to write "texts" in SMSB, have been mapped into 256 characters.
We have constructed two True Type fonts for SMSB, what results in two scripts with corresponding characters *SZ*(*S*tuttgarter *Z*eichen*s*atz) *Schwarzschrift* and *SZ Braille.* They are used in this paper and can be shown on the screen. On the Braille-output device, SZ Braille - characters appear if it uses the corresponding character-set max be in addition to other sets the user needs.
Assortment of the TrueType fonts SZ Schwarzschrift and SZ Braille,

▌▂▐ 〈 〉 = ≠ | + − ± √ ∈ ¬ → ← ∫ π ≈ ∑ ∅ ≥ ≤ <

A keyboard-layout that allows to type a formula character by character in the defined notation facilitates the writing [7].

R5: SMSB is suitable for joint teaching and learning of the blind and the sighted.

The TrueType fonts SZ Schwarzschrift and SZ Braille show that co-working and co-learning between the blind and the sighted is possible, because the written communication in mathematics has become possible. Some text-processing systems include formula-editors that may be comfortable for sighted persons. However, they are unusable by the blind. The SMSB-packet includes a macro, which transforms SMSB-terms into the representation, to which the sighted are familiar.

4.2 Transforming SMSB - Terms into a Graphical Representation

We have designed a word macro to transform SMSB-terms into a graphical representation. It is implemented in Visual Basic for Applications. The macro reads the entered string character by character and branches out to one of its ten subroutines, which handle

- digits and letters, +, -, : ...,
- lowered and raised characters and strings (exponents and indices),
- roots and roots of degree n,
- fractions,
- sums, integrals, and products,
- brackets, amount, binomial expressions,
- matrices,
- determinants,
- straight lines and vectors.

When the expected closing character has been read, the program constructs the corresponding string for the input of the formula field of the Microsoft Formula Editor. When the end of the marked term is reached, the string for the formula-field is ready to be processed. The intended representation of the written SMSB- term is produced. As a result, all terms, that follow the rules of SMSB, can be converted into a graphical form. It is possible to supplement the macro and to alter only parts. This macro made it possible to write teaching materials for the blind and for the sighted in only one step. It is no longer necessary to produce it twice.

5. Concluding Remarks

Arithmetic, geometry, algebra are basic cultural capabilities like reading and writing. Tanks to Louis Braille, blind people gained access to written communication within the world of the blind. Modern technologies have opened the way to close the gap between the blind and the sighted. Until now it seemed, that the world of mathematics is still a world, which is only open for blind specialists. Though the computer facilitates integration, it has not been possible to give access to mathematics in an adequate way and on a large scale. In this contribution, necessary steps, which could help the blind to more general education, are shown.

Last but not least, I want to thank *Uwe Christian* for his co-operation in the development of the SMSB during his time as a student in my research group, especially during the preparation of his diploma theses. My colleague *Nicole Weicker* helped to finish this paper by her great interest in this work and by fruitful discussions.

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