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**DD**MATH  
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# **Blind students and mathematics: Practices in Germany**

# Table of Contents

<b>Table of Contents</b>	2
<b>Introduction</b>	3
<b>Inclusion and the current framework conditions</b>	4
Inclusion figures in Baden Württemberg, Germany	4
Equal targeted schooling	4
Equipment used by blind students	5
Writing systems	5
<b>Learning the 8-point system in elementary school</b>	7
<b>Practices at the secondary schools</b>	10
Mathematics fonts	10
The LaTeX concept	10
First Rule	10
Second rule	11
Third Rule	12
Fourth rule	12
<b>Methodical-didactical aspects</b>	14
Technical resources	14
Input help	14
Working techniques	15
Verbalization	17
<b>Working on the web</b>	18
<b>Conclusion</b>	20
<b>Acknowledgement</b>	21

## Introduction

In Germany, the development of education for blind and visually impaired students has succeeded in giving alternatives to blind students either to visit dedicated schools for blind and visually impaired students or to join the general schools with the support of experts from the schools for blind and visually impaired students. These two alternatives have been established in parallel with systems that work in both school locations.

Within this report examples are mainly taken from schools located in the state of Baden-Württemberg. Almost similar approaches with minor differences are also reported in other parts of Germany.

Concerning the general schools the greatest challenge was to involve teachers teaching blind students without a degree in special education and also without lengthy further training. The personal computer has played the central role as a bridging technology. With the development of the Internet and the ever-increasing digitalization of our society, another aspect has been added in the last decade: many materials in the classroom are taken from the Internet by teachers or Internet content is presented directly to the students. Accessibility in this area is accordingly indispensable for the participation of blind and visually impaired students in the classroom.

In an educational context, not only accessibility is crucial, but usability is as important as well. For example, mathematical formulas that are read aloud with the speech output are accessible without barriers. If these formulas however are to be used for calculations in the classroom, then it is essential that they are readable via the Braille display. In Germany, importance has been placed mainly on readable mathematical writing under the fingers. Concerning this, there is an agreement for almost all teachers for blind and visually impaired students.

On the other hand however it is surprising that in English-speaking countries mathematics in the classroom is largely done on paper, although the relevant screen reader manufacturers come from the USA. Despite this, there is till now, no system that makes it possible for mathematics to be accessible on web pages in the sense of satisfactory and accessible usability. For example, it is not possible to copy a formula from a web page and edit it in an accessible and usable way in a word processing program.

Therefore, there is an urgent need for experts from different countries to work together in the field of education for blind and visually impaired students and develop common standards.

## Inclusion and the current framework conditions

Before we take a closer look at the mathematics system, we would like to briefly discuss the framework conditions in relation to the inclusion of blind students.

### Inclusion figures in Baden Württemberg, Germany

According to the 2019 State Government Report: " Weiterentwicklung sonderpädagogischer Bildungs-, Beratungs- und Unterstützungsangebote" the numbers of blind or partially sighted students were as following:

- 815 Students with special focus on vision at general schools: taught and accompanied by the special education service
- 961 Students at the "special education and counseling centers" (SBBZ) in the special focus of vision (educational programs - elementary school, lower secondary school, middle school and the special educational programs "learning" and "mental development")

If one assumes that in the SBBZ (special education and consultation centers)with focus on vision several hundred students are assigned to the educational programs "mental development", then it can be assumed that there is an inclusion rate of 2/3 or more for students who receive education either through educational programs or at the secondary education and vocational schools.

### Equal targeted schooling

The education of blind students at general schools is usually equally with all the other students target oriented. Markus Lang writes about the didactics of teaching visually impaired students, quoting Rudolf Schindele that no changes need to occur on the level of guiding and indicative goals (e.g., achieving cultural, vocational, and social competence) and on the level teaching content, the specifications of the general education plan should be considered binding<sup>1</sup>. As far as it concerns the media adaptation and learning environments Lang/Schindele demands optimally adapted aids and illustrative media". The selection of the optimal writing system must take into account not only the optimal conditions for the blind student himself, but also the conditions and necessities of the student's environment. Thus, practice shows that it is essential to take into account the competencies and resources of the teachers when designing the

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<sup>1</sup> Markus Lang, Didaktik des Unterrichts mit blinden und hochgradig sehbehinderten Schülerinnen und Schülern, Band 1: Grundlagen, Kohlhammer, 2008, p. 160f)

learning environment. For example, if learning a writing system is too burdensome for the educational environment, student-teacher communication will suffer and subsequently the quality of instruction will be affected.

## Equipment used by blind students

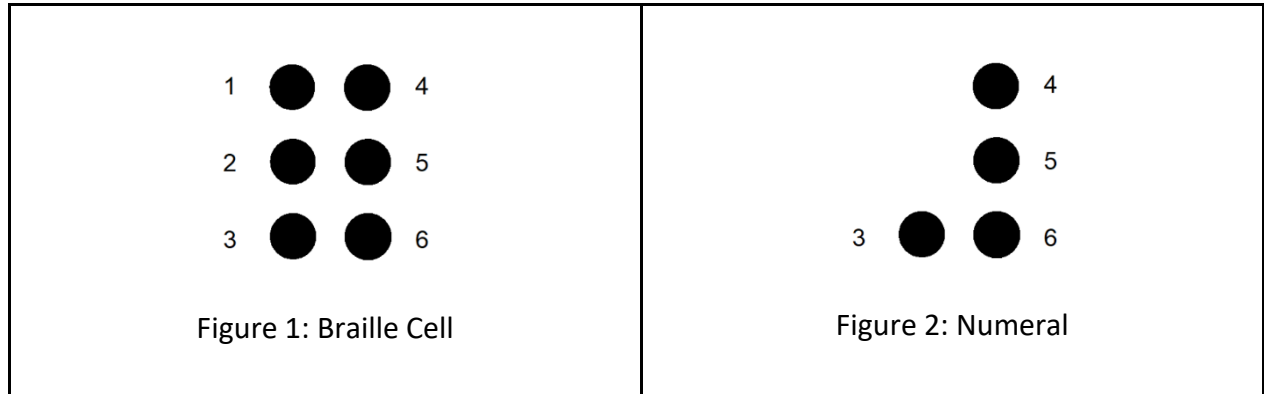
Equipment from the first grade includes a traditional 8-point typewriter, a Braille display, a PC with screen reader and Microsoft Word word processing program. The students are already familiar with the 8-point typewriter from preschool, where they playfully handle the machine and, without any pretense that they will learn to read or write, are simply playfully confronted with braille - much like their sighted classmates in preschool also are when they see writing in their everyday lives.

Textbooks are made available digitally and accessible by the so-called media centers in Germany in accordance with a standard agreed throughout Germany, the so-called e-buch standard (e-book-standard). The e-buch standard was developed in 2007 by a working group on behalf of the Media Centers Working Group of the Federal Expert Commission for the Review of Teaching and Learning Materials for blind students. It aims to ensure the quality of electronic teaching materials - especially the transfer of textbooks - at a standardized level and to facilitate the nationwide exchange and multiple use of these documents through a uniform design. Textbooks which are transferred into a digital format (.docx) are to be typeset according to the e-buch standard agreed upon by the media centers in all federal states using LaTeX as the mathematics font. The goal is not to compile into a two-dimensional representation, but to provide a notation that allows blind students to work effectively and safely while being easily accessible to the sighted teacher. (see LaTeX concept)

On site, the system is supported by educators specialized on the blind and visually impaired through regular visits (approx. 1 x per week). The teachers of the general school are assisted by a person responsible for the accessibility of the daily materials used by the teachers. If necessary the training and supervision of this assistant is also carried out by the special educators.

## Writing systems

The basis of all writing systems currently used in Germany is the braille system developed by Louis Braille in 1825, consisting of a cell with 2 columns and 3 lines. Due to the fact that only  $2^6$  different dot combinations can be represented with these characters, i.e. 64 characters can be generated, it was necessary to work with announcement characters and a double use of dot combinations for numbers, capitalization, music notations and various mathematical expressions, among others.



However, this practice led to problems when computer use was introduced. The computer offered for the first time the possibility of a direct, simultaneous translation from black letters on the monitor to braille on the braille display, which promoted and only really made possible a joint education of blind and sighted students in general schools. Of course, it is also possible to write at the PC in 6-point braille via braille input, but then sighted teachers without a sound knowledge of braille mathematics need additional translation in order to read (see table).






For a parallel display in braille and blackletter, a clear character assignment was needed, which was not possible with the 64 characters. After extensive debates and various tests, the 8-dot system "Eurobraille" or "Computer braille" has become accepted in Germany for digital work.











Black-letter	In words	6-dot mathematical font as Braille character	6 point math font in black on the screen
218	Two hundred and eighteen	⠠⠠⠠⠠⠠⠠	#bah
$\frac{2}{3}$	Two thirds	⠠⠠⠠	#b:
$(a + b)^2$	a plus b in Brackets squared	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	2'a +b`0;

## Learning the 8-point system in elementary school

In Baden-Württemberg, 8-point mathematical script is started from early education onwards. The students do not start learning handwriting on the PC. Printouts and handwriting production on the typewriter are done in 8-point. Thus, there is no hurdle when switching to the computer and working with the braille machine for written arithmetic is even possible with the 6-point typewriter without any problems, since all numbers and arithmetic signs relevant in elementary school are composed in between the first 6 points. One goal of inclusive education is to stay as close as possible to the didactics and pedagogy of the sighted classmates in order to avoid separation. For this reason, it has proven to be very helpful to start with a system in 8-point writing.

Here, one character in blackface corresponds to one character in braille. The reading learning process is accordingly comparable to the reading learning process of the sighted classmates. In grade one, when the letters are introduced, the blind students also learn the letters in Braille. The lowercase letters look the same as in the 6-dot system, and the uppercase letters have the dot seven added to them. Digits are also represented with only one character. Each digit is distinguished from the lowercase letter by the dot six. (see table)

Characters	8-point	on screen	6-point	on screen
1	1	1		#a
2	2	2		#b
3	3	3		#c
4	4	4		#d
5	5	5		#e

6	6	6		#f
7	7	7		#g
8	8	8		#h
9	9	9		#i
0	0	0		#j
+	+	+		+
-	-	-		-
-, * (multiplication)	*	*		!
÷, :, / (division)	:/	:/		:
()	()	()		2`

Therefore, it is not necessary to precede the announcement sign with letters or numbers. In everyday teaching at general schools, this facilitates the teaching considerably, because the sighted classmates also only have to learn one sign when a letter or a digit an A is introduced

After about six months, when the sighted classmates learn Braille, the PC is introduced for the blind students. The production of text is identical to the Braille typewriter because the Braille



display that is used has an 8-point input capability. In addition, over time, students learn to open a file on the PC , give it a name and save. They have a folder Schoolbag in which there are subfolders for each subject and in this subfolder there are then the files Schulheft.docx und Schulbuch.docx (corresponding to their notebook and their book)

Especially in mathematics, the communication between the blind student on one side and the general teacher of mathematics on the other side is easily possible. With the bridging technology of PC or Word documents and the presence of the black letter keyboard, the teacher can very quickly help the blind student to learn mathematics without having to go into the depths of the braille system. And that is one central point of the concept of success in the inclusion of students with the special focus of vision: An easy-to-learn system that is not only convenient for the student but also for the staff, and here especially by the general education math teacher, who can work in most cases directly with his student in less than 5 minutes of placement time.

On the other hand, a system that requires a lot of training - like the 6-point math fonts - leads to exclusion and separation in practice. So far there are no reported students in inclusion having to be taught mathematics in an external remedial class. Something that is probably common practice in English-speaking countries. The advantage of the shorter presentation does not outweigh the somewhat more compact presentation of the 6-point math scripts (e.g., Marburger Systematics, Nemeth Code, or even the newly developed UEB Math Code), in inclusive settings.

During the elementary school years, every necessary special character for math plus minus times divided is present on the keyboard and has one character in the 8-point system. Thus, even the 8-dot system in elementary school math is more compact to read than the 6-dot systems.<sup>2</sup>

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<sup>2</sup> In some federal states it is still common in lower grades to work in the subject mathematics with the 6-point mathematical script and a printout on paper. Here the translation is taken over by media centers and only later LaTeX is introduced.

## Practices at the secondary schools

With the transition to secondary school, students learn two additional systems in order to increase their working speed. First, they are given the opportunity to learn the traditional 6-point Braille systems to read faster in Braille. Secondly, the students are provided with a single-sided headset so as to be able to use the speech output on the PC and especially listen to longer texts. A comparison of the "reading speed" between very good braille readers and well-trained speech output users makes clear that second is more crucial in inclusion. Especially since it is very time-consuming to produce shorthand texts and the time resources at schools are limited.

## Mathematics fonts

With the introduction of fractions and root signs, the need to introduce LaTeX begins at the secondary level. One advantage of LaTeX is that only a few additional characters need to be learned. Rather, similar to a foreign language, new vocabulary, certain commands must be learned.

## The LaTeX concept

LaTeX offers, by its linear representation in the source text, good conditions for the representation on the Braille display. Since it is not a matter of having a final compilation of a text, the introduction of a mathematical environment or also other layout options can be completely rejected. The working group of the media centers has agreed on a concept for the school area, which defines the most important basics for the transmission. This concept can be found at <https://augenbit.de> and is based on fairly simple basic rules. These rules are shortly described as following

### First Rule

If the baseline is not left and the required mathematical character is available on the computer keyboard, these characters are used. The characters for multiplication and division will be represented by the characters \* and / or : which are also common in the sighted world.

For better readability, the principle of placing a space before each arithmetic character has become established.

Characters	8-point
[]	
{}	

### Second rule

If the baseline is left in a mathematical expression, other notations come into play.

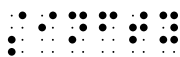

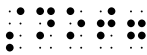
Superscripts are indicated by the circumflex "^", subscripts by the underscore "\_". If more than one character is superscript or subscript, the entire expression is written in curly brackets.

Example	8-point	on screen
$4^2$		4^2
$x_1$		x_1
$x^{a+b}$		x^{a +b}
$x_{t+1}$		x_{t +1}

### Third Rule

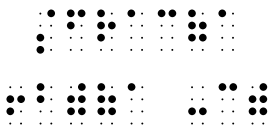
In addition to the characters that can be found on the keyboard, single characters such as the sign for infinity, unequal or an arrow are needed in mathematics.


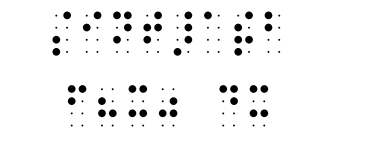
These are entered with the LaTeX name and the corresponding backslash.

Example	8-point	on screen
$\infty$		<code>\infty</code>
$\rightarrow$		<code>\to</code>
$\neq$		<code>\neq</code>

### Fourth rule

Beside these expressions there are notations, which have a two-dimensional representation in themselves. These include, for example, fractions, roots, integrals and matrices. These are also announced by a command word and converted into LaTeX notation. Small fractions with only one character in numerator and denominator can be written with the slash (e.g. 1/3)

Example	8-point	on screen
$\frac{a+b}{a-c}$		<code>\frac{a+b}{a-c}</code>

$\sqrt{2+x}$		<code>\sqrt{2+x}</code>
$\int_a^b f(x)dx$		<code>\int_a^b f(x) dx</code>

Here again a short overview how the LaTeX applied in the schools differs from the standard LaTeX:

- Permissible notation of small fractions (one-digit numerator and denominator): 1/2
- Multiplication sign: \* instead of `\cdot`
- Each arithmetic character ( +-\* / ) is preceded by a space for better readability
- The backslash is omitted for curly and square brackets or the percent sign.
- Shortening is possible (but not used in textbooks) -> Shortening table

## Methodical-didactical aspects

Didactics in mathematics is strongly oriented towards the spiral principle. The mathematical content is worked on at an increasingly higher level in the course of the school career. Existing knowledge is caught, correlated and further deepened. This can be compared with the mathematics notation. In elementary school, only numbers and arithmetic symbols are needed at first. First fractions can be implemented with the slash notation ( $1/2$ ). In the secondary level, exponents and other fractions are added as well as special notations such as periodic numbers or number sets. Gradually, the mathematical vocabulary is expanded and together with it the LaTeX expressions. Thus, there is never an excessive demand due to many new notations.

An overview of the required expressions in secondary education can be found at the following URL: <https://augenbit.de/wiki/index.php?title=LaTeX-Manual-Sekundarstufe1>

Accordingly for the upper secondary education on this link:

<https://augenbit.de/wiki/index.php?title=LaTeX-Manual-Sekundarstufe2>

## Technical resources

Blind students in Germany usually work with Microsoft Office and the screen readers JAWS or NVDA. Microsoft Office offers good possibilities to work effectively with the mathematics font LaTeX. It also has the advantage that it is well adapted to screen reader use and is one of the most commonly used softwares in the professional world for spreadsheets and word processing. Thus students, by finishing with the school, bring with them into the workplace a sound knowledge of how to work effectively. This can be a crucial building block for a successful career.

## Input help

The menu ribbons for Microsoft Word, which were developed by the Media Advisory Center Ilvesheim, are a help. The LaTeX menu ribbon is used for textbook transfer by the media centers, worksheet transfer by the assistants and by the students themselves.



**Figure 3** LaTeX ribbon

Via menu entries all commands relevant in the school can be inserted directly, so that expressions like  $\frac{\{ \}}{\{ \}}$  do not have to be typed, but are inserted via a keyboard command.

The cursor is then already in the first bracket, so that the notation of the counter can be started directly.

The commands can also be easily integrated into the toolbar for fast access, ensuring particularly fast access to certain commands depending on the topic. In the textbooks, students are given LaTeX in the elaborated form - the ribbon allows switching between the long form and a defined number of abbreviations. For example, the fraction notation can be shortened to  $\frac{a}{b}$ , which is interesting for the students due to the limited character capacity on a Braille display.

It is important to note that students must be able to read LaTeX in the long form defined above. The students often use simplified forms in the notation, for which it is decisive for the correction whether the mathematical correctness is maintained. Notations based on the notation in Excel or Maple can be used as orientation.

## Working techniques

In order to be able to handle complex expressions with confidence, certain working techniques and habits are very helpful in addition to the ability to enter them quickly. These need to be established at an early stage. These include effectively copying and pasting entire lines or systematically examining parentheses. The ability to work with shortcuts makes it easier for students to work confidently in Microsoft Word.

Students receive their textbooks currently in one or more Word files (one file per chapter) and copy and paste assignments into their math notebook in class to work on. With the help of various marking techniques (F8 marking mode or marking via the outline view), the tasks can be specifically extracted in their entirety.

Over many years, numerous exercises have been developed for these working techniques, which enable particularly effective handling of documents in the e-book standard and which are made available to colleagues via [augenbit.de](http://augenbit.de).

As an example, a fraction equation will be used to show how systematic copying, pasting and editing can also be used when working on a task.

$$\frac{x+4}{x} = \frac{16}{x^2-4x} - \frac{4}{x-4}$$

1. Copy the 1st line and paste it in the line below, write "Denominator:" in front of it and delete everything that is not a denominator.

Denominator:  $\{x\}{x^2 - 4x}\{x - 4\}$

2. Copy the whole line again and paste it in the line below, write "Factors:" in front of it, decompose each denominator (inside the curly brackets) into factors

Factors:  $\{x\}{x * (x - 4)}\{x - 4\}$

3. Copy factors line and paste in the line below, write "HN:" in front of it, make the first denominator the main denominator, check: which factors are missing in the first bracket compared to the 2nd and 3rd brackets (2nd and 3rd denominators)? Add \* into the first denominator, when 2nd and 3rd denominators are checked, delete them.

HN:  $x *(x - 4)$

4. Look at the denominator (best factors row) and consider what may not be used for x, define a set.

$\mathbb{D} = \mathbb{Q} \setminus \{0; 4\}$

5. Continue working with the original equation

$\frac{x + 4}{x} = \frac{16}{x^2 - 4x} - \frac{4}{x - 4}$  | Extend to HN

6. Copy line and paste it in the line below

$\frac{(x + 4) * (x - 4)}{x * (x - 4)} = \frac{16}{x * (x - 4)} - \frac{4 * x}{(x - 4) * x}$  | \* HN, Shortening

7. Copy line and paste it in the line below

$(x + 4) * (x - 4) = 16 - 4x$  | Unravel parenthesis



8. Copy line and paste it in the line below

$$x^2 - 16 = 16 - 4x \quad | \quad -16 + 4x$$

9. Copy line and paste it in the line below

$$x^2 + 4x - 32$$

10. Formulate pq formula and insert values for p and q

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

$$x_{1,2} = -\frac{4}{2} \pm \sqrt{\left(\frac{4}{2}\right)^2 + 32}$$

11. Prepare for Excel, copy and transfer result to Word

$$x_1 = -\frac{4}{2} + \text{WURZEL}\left(\left(\frac{4}{2}\right)^2 + 32\right) = 4 \quad | \text{not in D!}$$

$$x_2 = -\frac{4}{2} - \text{ROOT}\left(\left(\frac{4}{2}\right)^2 + 32\right) = -8$$

$$L = \{-8\}$$

## Verbalization

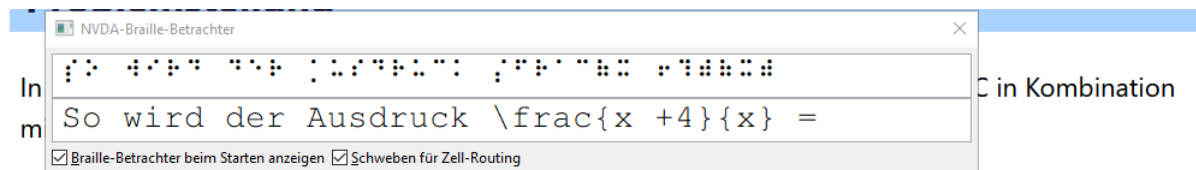
For the students, not only their own working techniques are important, but also clear language and designation by the teacher. Parentheses must be clearly labeled with their beginning and end, and the beginning and end of the numerator and denominator of fractions must also be clearly labeled, so that all students can follow, whether they are using the LaTeX expression or the two-dimensional figure.

## Working on the web

Currently, working in Microsoft Word with student books as docx files is the standard in Germany. However the Covid Pandemic has also increased the focus on the web area.

Teachers have had to transport materials via the Web during school closures, and textbook publishers as well as other vendors have significantly expanded their offerings. Unfortunately, accessibility is usually still very neglected here. However, solutions are also quite possible for the web area!

In HTML, MathML is defined as the standard for mathematical expressions, however this has not really caught on. Since the Chrome browser stopped supporting the representation, the pure MathML implementation continues to shrink. In the meantime Mathjax has become the unofficial standard on the web as a rendering option via a Javascript package. The rendering is possible in different outputs and also the input offers many possibilities. Input in LaTeX, MathML or ASCII-Math is allowed, output in HTML-CSS, SVG or also MathML. LaTeX is used for most formulas and if this is the case, the expression can be "extracted" via an additional script and printed on the line.



Da in der Brailleschrift keine Abweichungen von der Grundlinie möglich sind, ist es erforderlich, Ausdrücke wie Wurzeln, Hochzahlen, Brüche oder auch Integrale in eine lineare Form zu bringen. Im deutschsprachigen Raum hat sich im digitalen Arbeiten LaTeX als Mathematiksschrift etabliert.

So wird der Ausdruck  $\frac{x+4}{x} = \frac{16}{x^2-4x} - \frac{4}{x-4}$  von den Schülern im Quellcode gelesen und bearbeitet.

### Figure 4: Representation on the web

We have set this up on one page as an example and are in close exchange with the developers to be able to implement this output without an additional script.

In addition to text and vector-based 2D graphics, MathJax provides the option to deliver MathML in the background. This expression can be explored by the math viewers of the screen readers in a speech-based way. After entering the formula, one can navigate through different levels using the arrow keys and thus, for example, view only the numerator or only the denominator of a fraction.

Since probably the textbook of the future will be html-based or at least much of the content will be provided html-based, it is very important that the needs of students are seriously taken into account. With MathJax and a LaTeX source code input and associated LaTeX output on the line, this can potentially succeed.

## Conclusion

The current report is mainly based on practices applied in Baden Württemberg but also reflects the situation and offerings to blind and partially sighted students within Germany

With the methods presented in this report blind students reach their university entrance qualification. Beyond that they can continue to apply the learned methodics also at the university and even later by writing their dissertations or diplomas with LaTeX. The big criticism or fear of the representatives of the established 6-point mathematical font that characters in 8-point Braille should not be readable or that LaTeX would be too long, have rather not been confirmed so far in everyday life at the schools.

It should be admitted that this methodology might have weaknesses and can certainly be further developed. One possibility can be the use of functions known from other systems such as Lambda. Furthermore, the question of how screen readers display mathematics on the Braille display when it comes to interpreting formulas in web pages is still open and needs research to be made in the future.

## Acknowledgement

For this study we would like to acknowledge the valuable support and information provided by Michael Schäfler and Barbara Henn from the Schloss Schule Ilvesheim.