ANALYSIS OF THE STATE OF THE ART

Best practices and development prospects

Intellectual Output 01

JULY 2021

PROGETTO DDMATH

Digital learning in mathematics for blind students

ERASMUS+ Program

Co-funded by the Erasmus+ Programme of the European Union

DDMATH

ERASMUS+ Program

DDMATH PROJECT

Digital learning in mathematics for blind students

INTELLECTUAL OUTPUT 1

ANALYSIS OF THE STATE OF THE ART, BEST PRACTICES AND DEVELOPMENT PROSPECTS.

July 2021

Project within the ERASMUS+ program – School Education Gateway – Action KA2 - Strategic partnerships for digital education KA226 – Convention n. 2020-1-IT02-KA226-SCH-09557

Project number: ERASMUS+ program – School Education Gateway – Action KA2 - Strategic partnerships for digital education KA226 – Convention n. 2020-1-IT02-KA226-SCH-09557
Project title: DDMATH- Digital learning in mathematics for blind students
IO type: Publication
IO number 1
Contractual date of the Delivery: July 2021
Actual date of the Delivery: July 2021
Title of IO: Intellectual Output 1 Analysis of the state of the art, best practices and development prospects
Nature of the Deliverable: Public
Authors: All partners of the DDMATH Consortium
Abstract: Analysis of projects, products, systems and codes created for, or used by, the blind for writing mathematics on the computer in various countries in the world, with special reference to what may potentially contribute, directly or indirectly, to the DDMATH project.
Keyword List: Blind, mathematic, Braille, 8dots, education, computing, LaTeX
The European Commission's support for the production of this publication does not constitute an endorsement of the contents, which reflect the views only of the authors, and the Commission cannot be held responsible for any use which may be made of the information contained therein.

Table Of Contents

Index of Figures

1 Introduction

The first phase of the DDMTAH project sees the creation of a state-of-the-art analysis document on the subject of scientific studies for blind students, in order to guarantee the teachers concerned, the students and their parents, a full understanding of the existing solutions in terms of didactic and methodological proposals, existing technological solutions, existing digital material in the specific context of integrated and inclusive digital teaching (i.e. with the active participation of all students), which can also be adopted in the composition of virtual classes.

The usefulness of this study is to collect the best practices and the innovative and positive experiences conceived by the school world, the research trends in the field of technological aids, to know the most relevant stakeholders, in order to disseminate them to offer elements for reflection to promote the future experimentation, to offer elements of reflection for the elaboration activities of the second phase of the project, that is the implementation of the didactic resources and video guides ready to be used directly by the teachers.

Therefore, the objectives of this report are to define a conceptual and theoretical framework that allows to describe the reference models at national level, based on tested and widespread tools and aids, and once identified and described the dimensions and key elements of the models, define the essential preconditions and success factors that can guarantee the effective implementation and development of the subsequent activities of the DDMATH project.

From an operational and methodological point of view, to implement this IO1 a series of actions have been launched and among these:

- ✓ a comparative analysis of scientific publications and research on the models of teaching mathematics to the blind;
- ✓ the recognition of good practices implemented at national level deriving from the results of the analysis of the user needs of IO2;

- ✓ a thematic workshop (which took place on 13 November 2021 in Verona) for in-depth analysis with the project partners (analysis of the information collected for the definition of strategic choices). The workshop also carried out the elaboration of a hypothesis of a feasible model at European level and of integrated digital teaching for blind young people, with the definition of the essential requirements to guarantee the concrete application and development of an inclusive training path.
- ✓ the characteristics of the existing IT solutions, i.e. a systematic and complete overview of the existing and available resources in the specific field of access and study of mathematics that can also be used in a context of DDI for blind students at European level.
- ✓ The literature concerning the most important theoretical models in the field of mathematics teaching, considering the issues of access through the channel of touch and / or hearing and the conceptualization of signs, which for some encodings correspond to the description of the graphic aspect of a formula (this requires the reconstruction of a mental image of the graphic formula), while for others, as in Lambda, it is a content-oriented code.
- ✓ specific existing and adopted 6 and 8-dot mathematical codes,
- ✓ existing mathematical editors in the field of accessible mathematics (in Braille mode, spoken mathematics, or based on other standard codes such as LaTeX),
- ✓ software for conversions from traditional format to digital braille (and vice versa),
- ✓ analysis of the state of the art for tools and solutions specifically dedicated to the visually impaired and solutions specifically dedicated to "doing mathematics" for children with severe motor disabilities, and dyscalculics.

The analysis of the state of the art also included products only marginally connected to the DDMATH project, in particular those used for the conversion and preparation of texts and for Braille printing (such as Labrador, Bramanet, Italbra) or the production of read-only mathematical documents that cannot be managed by blind people (such as Dotplus), generic software in the scientific field for other disabilities or other special needs such as for children with dyscalculia. The research activity also took into consideration models, proposals and solutions that have not been further developed over the years and in many cases are no longer used today, but which have played a significant role in subsequent research.

For the presentation of the existing solutions and experiences in the field of products in the scientific field for children with dyscalculia or other SEN, a presentation in the form of an easy-to-read card was used, and therefore it was considered useful to adopt a model already extensively tested in the project of the CNR of Genoa called Essediquadro (1999-2017).

Such a support for the project was made possible as the consortium was joined by the CNR of Genoa as an associated partner, within which there is an ITD center, i.e the Institute of Didactic Technologies. The center of didactic technologies deals with selecting and experimenting computer programs and proposals based on didactic technologies.

The Essediquadro data sheet proposes to analyze the experiences and the products made in the light of four considerations.

After providing general information (who developed the system, when and what for), the innovative aspects are described, with brief history and current status.

The limitations of the product are then described (what it doesn't do) with any difficulties (technical, didactic, economic, operational…) which may restrict dissemination. This section is filled in using the assessments (or self-assessments) present on the web, integrated by software tests (if possible) and code tests, plus a comparison between the product features and needs expressed by users on the basis of IO2 (users' needs).

Lastly, the product is analyzed in the light of the aims of the DDMTH project to check if there are any overlapping areas, instruments or experiences that can be shared to optimize the work, validating the experience so far and its results.

Considering the dependence between users' needs and existing experiences and solutions, it is clear as a consequence that this work completes and is closely linked with the one of I / O2 on users' needs.

Obviously, as regards projects and proposals that are no longer developed or available, they are only presented in general form, trying to grasp the elements that characterized their success and that inspired subsequent research.

Indeed, the questionnaire delivered to students and teachers contains a number of questions concerning the use of alternative systems for computer mathematics notation: special software, generic software adapted to particular needs, codes for mathematical notation, and so on. This was an attempt to find out what other work had been carried out in the field, where no traces were to be found in the Internet.

An important part of the analysis of the state-of-the-art was the study of conventional 6-dot (hence not IT) Braille mathematical codes commonly used in Europe.

The codes which have been examined so far:

Italian Braille code

English Braille code

French Braille code

Spanish Braille code

Russian Braille code

Portuguese Braille code

The Marburg code (also adopted in Poland)

Nemeth code

2 Mathematical Braille notations

2.1 Background: Braille mathematics notations

The Braille notation, as invented by Louis Braille for the representation of text, is based on cells of six dots. This makes it possible to represent 63 distinct symbols (because 2^6–1 = 63). That is more than enough to represent 2^6 alphabetical characters plus space and punctuation – as found in the sort of literary text that Braille was endeavouring to represent. The fundamental problem of representing mathematics in Braille, though, is that mathematics uses a lot more than 63 symbols.

2.2 Braille fundamentals

The Braille representations of the letters of English Braille are shown in Figure 1. Note that there are some variations between different literary Braille codes, even between countries which use the same alphabets. For instance, in French – the language of Louis Braille – there is no letter w and so a Braille code had to be added to Braille to represent English text.

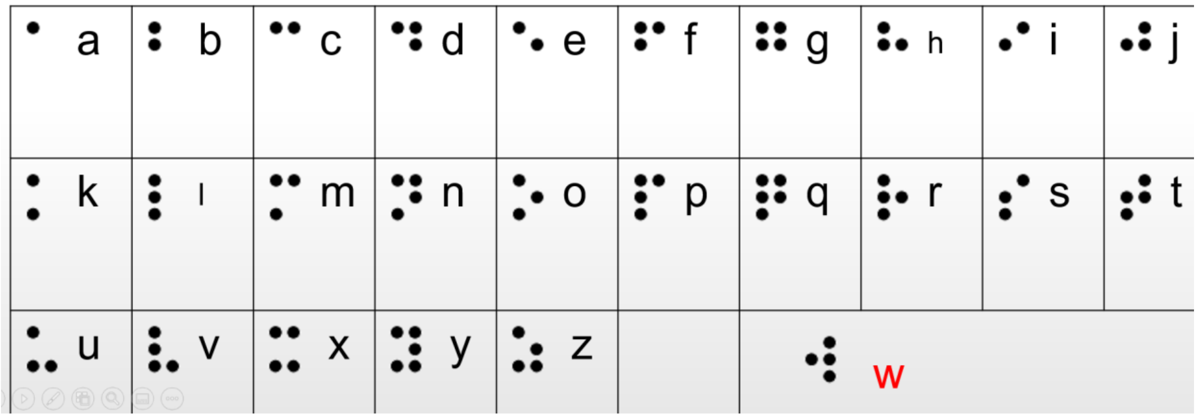


Figure 1: The Braille representations of the letters

In order to describe Braille cells there is a system of numbering the dots, as in Figure 2. Thus, A can be referred to as being represented by dot 1, B by dots 1 and 2, C dots 1 and 4 and so on.

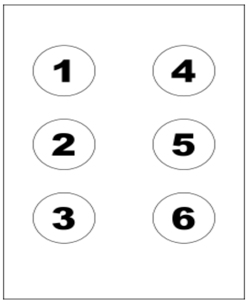


Figure 2: Numbering of the 6 Braille dots

So, the letters of the alphabet are represented using 2^6 of the 63 Braille codes. It is significant to note that in standard Braille the 10 digits do not have separate representations. Rather, they are represented by the same cells as the first 10 letters of the alphabet, preceded by a special cell (the 'number-sign') signalling that the following cell or cells should be read as numbers, not letters. So, the digits are represented as in Figure 3.

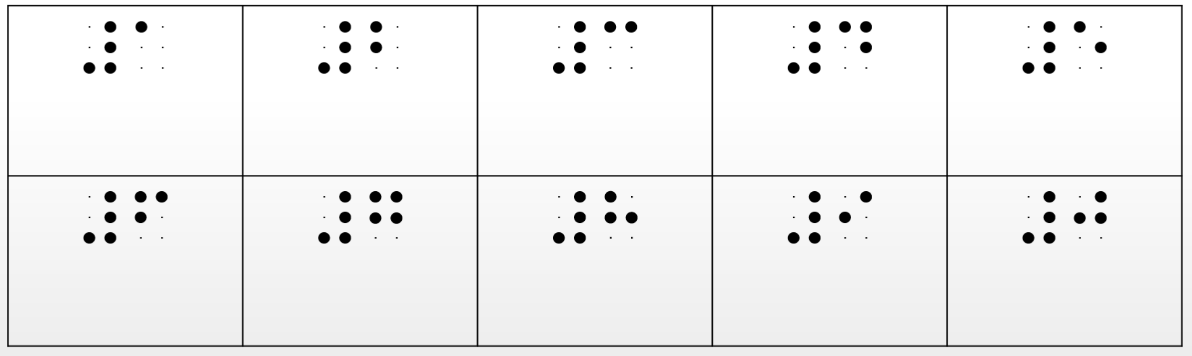
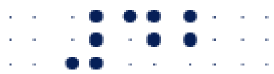


Figure 3: Braille representation of digits

Multiple-digit numbers are preceded by a single number-sign, so that for instance 42 is



By using combinations of cells, it is possible to represent a much greater range of symbols. In principle, two-cell combinations can represent 64^2-1 = 4095 symbols. However, it is not that simple. For a start, it is not convenient to have to read two cells to get one item of information. Furthermore, in practice not all combinations can be used unambiguously.

Another approach to extending the range of symbols that can be represented in Braille is to extend the Braille cell to eight dots. This is common on computer Braille displays. With eight dots, 2^8-1 = 255 symbols can be represented. (The numbering of the eight dots is shown in Figure 4) This is more than enough to accommodate all the symbols of most mathematical representations. However, there are problems with 8-dot Braille. The main one is that most Braille readers have been taught the 6-dot code and it is not simple to re-learn an 8-dot code. It can also be difficult to switch between reading one code and another. The difficulty of this depends partly on the design of the 8-dot code, and the degree of similarity that it retains with the traditional 6-dot code. Indeed, there are a number of 8-dot codes in existence - and that is itself a problem. With no single standard, there is a question as to which code to learn and then, having learned one code, to have material available in a different code is useless.

1	4
2	5
3	6
7	8

Figure 4: Numbering of the dots in 8-dot Braille.

Visual mathematical notations are not simply sets of additional symbols. Rather, they make the most of the two dimensions of paper, but Braille is a strictly linear notation. Visual notations have been developed such that they embody some of the mathematical properties visually. To take a simple example, an expression such as

$$2x + 3 \tag{1}$$

combines the mathematical operations of multiplication and addition. The multiplication (of 2 and x) is implicit with no intervening operator, whereas the addition operation is signalled by the + sign. Yet, notice that the 2 and x are printed close together, while there are spaces around the +. This reflects the higher precedence of the multiplication operation: the 2 and x must be multiplied and then their product is added to the 3.

Another example is the fraction:

$$\frac{x+1}{x-1} \tag{2}$$

This clearly is a compound entity with two components, the numerator and denominator. Linear notations are available, such as

$$(x + 1) \div (x - 1) \tag{3}$$

but this does not capture the mathematical properties as clearly – and it requires additional symbols (the parentheses) in order to avoid ambiguity.

Another consequence of the linearization of notation in Braille is that yet more symbols are required. For instance, the vertical position of a symbol may be significant in printed notation, such as a superscript for a power, as in x^2. To represent that in Braille it will generally be necessary to introduce a new symbol representing the operation of raising to the power, such as x↑2.

In considering the design of Braille codes and Braille representations of information, it must always be borne in mind that Braille perception is very different from visual perception. Principally, at any time, the reader can only sense whatever is under one or two fingertips. Thus, for instance, it is very difficult to get an overview of anything that is large or complex. Similarly, it is difficult to make a connection between two entities that are spatially separated. For instance, in a table of values it is visually easy to see the connection between entries at opposite ends of a row (e.g. the row label and an entry in that row), whereas in Braille it is impossible for most Braille readers to read both entries simultaneously and difficult even to be sure that they are in the same row.

Another important difference between Braille and print is simply the relative sizes. A 6-dot Braille cell occupies approximately 5.8 x 10.0 mm (including the necessary empty space around it), whereas an average letter in 12-point print typeface occupies approximately 4.9 x 1.8 mm Thus, even if one can achieve aon-to-one mapping from text to Braille, the Braille will occupy approximately 6.5 times more space.

2.3 Learning mathematics in the presence of low vision or blindness

Learning mathematics for visually impaired and blind people is a great challenge, not so much because of the impossibility in itself, but because of the lack of suitable tools.

In the early years of school, the learning of basic concepts of arithmetic and geometry occurs mainly through the constructivist methodology through the use of manipulable materials or approaches such as multisensory (space-time).

The virtual manipulables are suitable for the visually impaired (eg. Didax, Mathigon, Math Learning Center) as well as the mixed tangible and virtual programs (Play Osmo, Hands On Equations), and multisensory games (eg. Animal Watch, Math Melodies or web app such as We Draw).

When dealing with more complex concepts, two problems arise from secondary to university: the accessibility of materials and the choice of tools to operate with.

Regarding accessibility, there are various aids but they are not tools for everyone, due to the costs and difficulty of use. These include: Braille printers and keyboards, haptic tablets, embossed printers, text-to-speech (TTS), screen readers, OCR scanners, single-line mathematical language editors (e.g. Latex, MathML, Chatty Infty / Infty Editor).

Figure 5: Braille Printer, Upgradeable Braille Display, Embossed Printer[1]

To solve operations specific software can be used: calculators (normal, scientific, statistics), solvers of mathematical problems step-by-step or work environments that support the Braille code.

A big problem that may arise concerns the dialogue and translation from the language for the blind to the visual one, to have a discussion with peers and teachers, only possible with systems that have double windows and double output such as Lambda.

Other obstacles are the load of working memory and the ambiguity of mathematical notations (which are partially solved through Earcon, Spearcon and prosody) and the long learning curve of these languages.

[1]reworking of images found on manufacturers' websites

Finally, the Braille language is not universal (there are different standards in various countries such as Nemeth etc.), moreover the 6-dot Braille language is not sufficient to represent all mathematical symbols, so Lambda, for example, uses 8 dots .

Even without dedicated programs it is possible, within the Microsoft Office, Libre Office and I-Work suite packages, to read and write mathematical notations in Windows, Mac and Linux environments.

Compared to STEM (Science, Technology, Engineering, Mathematics), the attention to the blind is becoming more lively in recent years and the area of programming is also arousing a certain interest. Also in this case there are two approaches: Visual - Tangible (in the Windows world) with manipulable objects for the first years of school and Visual - Auditory (in the Apple world) with the support of Voice Over.

3 Traditional National 6-dot mathematics Braille codes

The collection of data and the analysis of national 6-dot mathematical Braille codes are aimed at proposing the project with a view to summarizing the proposals and solutions adapted to schools in the main European countries. A further investigation will be present in the next chapters which will examine the 8-dot Braille encodings (born as an evolution of the original 6-dot ones), created for specific use within computer systems.

A complete and updated description of the Braille codes used in the various countries of the world is available in the World Braille Usage Third Edition dl 2013 report downloadable from the following link:

https://www.perkins.org/wp-content/uploads/2021/07/world-braille-usage-third-edition.pdf

The national codes we acquired through partners are as follows:

	National code	6-dot	Authors	Year
1)	Italian	BIC (Bibiotca Italiana dei Ciechi)	Russo, Domenicali	1998/2003
2)	English	RNIB: Royal National Institute for the Blind	The Braille Authority of the United Kingdom	1987
3)	French	DTEA	Commission Evolution du Braille Francais	2001
4)	Spanish	ONCE	Comision Braille Española	1987
5)	Russian	LOGOS	VOS	1990
6)	Portuguese	ACAPO	ACAPO	1994
7)	Marburg code Adopted in Germany and in Poland	Deutsche Blindenstudienanstalt	Dr.Epheser, Dr.Pograniczn, Dr.Britz	1992
8)	Nemeth code	NBC	Dr. Abraham Nemeth	1973

Two considerations emerge on preliminary comparison:

There are huge differences in codes between different countries in rendering in Braille, not only in relation to complex structures but also elementary symbols (arithmetic operators, brackets), and even in numbers.

To render the composite structures (fraction, powers, roots and functions in general with elementary content) structure is generally organized in two ways:

- ✓ the use of dedicated opening and closing symbols for the function (e.g. open root, close root), in this case these are content codes;
- ✓ the use of “blocks” i.e. generic brackets which are always the same and close off part of a function. In this case they are presentation codes.

3.1 Italian Braille code

The study version was published in 1998, (and an upgrade was in 2003), by the “Regina Margherita” Library for the Blind. Chief editors are prof. G. Russo and prof. L. Domenicali.

All structures and leading mathematical symbols in each school level are present.

It is an unambiguous content code, somewhat prolix and with few abbreviations.

Here are the links where to find the manuals of the Italian 6-dot code

Page of the website of the Italian Library of the Blind, where you can download the manual in PDF and word:

https://www.bibliotecaciechi.it/il-nuovo-codice-braille-italiano

Table for printing in black of the mathematical Braille code 2003:

https://chezdom.net/wp-content/uploads/2008/09/codice-matematico-2003.pdf

Systematic presentation of the Cavazza institute:

https://www.cavazza.it/invitoalbraille/html/fr32.htm

3.2 English Braille Code (Unified English Braille)

In 1987 the Royal National Institute for The Blind published the English code drawn up by The Braille Authority of the United Kingdom. Based on this work The International Council on English Braille has developed a Unified English Braille which has also been adopted in Australia, Canada, Ireland, New Zealand, Nigeria, South Africa, United Kingdom, United States. The EPO Mathematical Committee is periodically active to improve the assignment of symbols for technical materials and any rules necessary for their use.

It is a code with all symbols and is unambiguous in content terms. To abbreviate or simplify some structures, there are a number of exceptions to the general rules and spacing is used frequently. This is simple for experts in the code, but very hard for automatic codifying. The document has many examples and symbols for physics and geometry are included.

Additional information:

Wikipedia article on UEB:

http://en.wikipedia.org/wiki/Unified_English_Braille_Code

3.3 French Braille code

The Commission Evolution du Braille Francais has updated (September 2001 and 2007) the Notation Mathematique Braille code of 1971. This code is also used in Madagascar and and partly in Portugal.

It is a presentation code which blocks are used to define functions. The blocks are used wherever in the hared copy there are no ordinary brackets. Some abbreviations, for example for goniometric functions, are interesting, with a single symbol instead of the translation of signs as in the other codes. Complete with units of measurements, vectors and geometry.

2001 Manual:

https://chezdom.net/wp-content/uploads/2008/07/braille-math-09-01-d1.pdf

2007 Manual:

https://chezdom.net/wp-content/uploads/2008/07/notation_mathematique_braille2.pdf

3.4 Spanish Braille code

The Spanish code was drawn up by the Comission Braille Española. Itwasapproved in June 1987 by the Representantes de lasImprentas Braille de HablaHispana in Montevideo (Uruguay).

A recent version is the one made by the Braille Code Unified Mathematics (CMU) for Spanish-speaking countries, made by the Spanish Braille Commission:

The manual can be downloaded from the ONCE page from this link:

https://www.once.es/servicios-sociales/braille/documentos-tecnicos/documentos-tecnicos-relacionados-con-el-braille/documentos/b5-signografia-matematica.pdf/view

As in the French code the structures which require an opening and closing sign do not use a unique symbol identifying the symbol and generic auxiliary brackets to close off the subject.

There are no symbols for geometry.

3.5 Russian Braille code

The All Russia Association of the Blind (VOS[2]) has published the Russian Braille mathematics code.

One version is available in English. This code is used in some European countries bordering the Russian Federation (Czechrepublic, Slovakia and other countries).

The bibliographic indication is as follows:

[2]https://www.vos.org.ru/ (2021)

System of Symbols for Math, Physics, Chemistry, and Astronomy: A Learning Aid, by A. G. Bykov, M.I. Egorov, F.B. Morozova and I.V. Proskurjakov. Edited by Proskurjakov and Bykov. Committee fr Exact Sciences for All Russia Association for the Blind. 2nd ed, rev and enl. The All Russia Association for the blind. Moscow, 1982.

It contains many examples, a content structure and dedicated symbols. There is a chapter on calculation and complex numbers and a highly detailed chapter on analysis with many examples and symbols for higher maths.

3.6 Portuguese Braille code

This code relies basically on the CMU "Codigomatematicounificado para a lingua castelhana" of 1987 with some variations and integrations connected to the traditional Portoguese and to French as well. The code is thus named:

CódigoMatemáticoUnificado para a Língua Portuguesa –CMU

It is used in all Portuguese-speaking countries.

The manual is available and traceable on various websites such as the following:

http://www.sbem.com.br/enem2016/anais/pdf/5413_2991_ID.pdf

It is a presentation code, in fact we estimate the use of auxiliary parentheses that are equal to encompass complex topics from functions (roots, fractions, potences....).

There are even abbreviations that are similar to those of the Italian code (simple fractions, square roots).

All the symbol and structures used in the secondary school, including Geometry, are described.

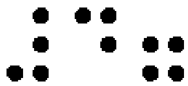
3.7 The Marburg code

The German language code International Mathematikschrift fur Blinde was published by the Deutsche Blindenstudienanstalt of Marburg in 1992 (last update).

It is a content code, well-defined and complete.

The InternationalMathematikschrift für Blinde is better known as the Marburg Notation. This is a six-dot notation which maintains compatibility with literary Braille. Thus, numbers do not have separate codings, but use the number-sign plus letter notation described earlier. This leads to complications in formulas which mix numbers and letters, in which a special symbol (dot 6) must be used to signal that the following cell is not a digit, but a letter.

The numerator of a simple fraction is written as the number (preceded by the number sign) and the denominator is the number lowered (i.e. dropped one row in the Braille cell). Thus, the fraction 4/7 is written as



but, confusingly, the equals sign (=) is also represented by a lowered g, so that the above might be interpreted as 4=.

It is used practically in all German speaking countries (Germany and Austria) and with some difference also in Poland, Denmark, The Netherlands, Norway and former Yugoslavia.

Additional information:

http://www.mediablis-bayern.de/downloads/marburger-mathe-neu-2013.pdf

Neufassung und vervollständigung des systems der internationalenmathematikschrift für blinde, by H. Epheser, K. Britz and F. M. Scheid. Marburg/L.: StändigeArbeitsgemeinschaft “Mathematikschrift”, Deutsche Blindenstudienanstalte.V. 1986

https://www.blista.de/sites/default/files/blista-blickpunkte-thema7.pdf

3.8 Nemeth code

The Nemeth Code is the standard Braille code for mathematics developed in 1946 by Dr. Abraham Nemeth.

Its creator, a blind mathematician, first used the code to record personal notes.

Since Dr. Nemeth's first publication, other authors have written books and instructional manuals on Nemeth Code.

Nemeth was accepted as the standard code by The Braille Authority of North America. In 1992 was integrated into Unified English Braille.

The Nemeth site also says:

“Nemeth Code is capable of rendering all mathematical and technical documents in six-dot Braille, including:

· Arithmetic

· column arithmetic including carrying and borrowing;

· long division

· Algebra

· Geometry (figure drawings not included)

· Trigonometry

· Calculus

· modern mathematics up to research level”

Nemeth Code can also be used in any scientific discipline that requires mathematical notation. Since Nemeth Code is a six-dot Braille, it can be generated using the Braille tools commonly available in North America, such as: slate and stylus, Perkins Braille Writer, or computer.

Addition information in:

https://en.wikipedia.org/wiki/Nemeth_Braille

Version of 1972: https://nfb.org/images/nfb/documents/pdf/nemeth_1972.pdf

4 Scientific notations via computer

The problem of the access by the blind to mathematical and scientific notations via computer has been tackled over the last twenty years in numerous countries.

An important role has been carried out by several blind University researchers and professors who tried to improve systems they sometimes created for their own use, making them available for other blind people, and improving the systems. Universities have used Internet for many years as a primary communications instrument, so this experimentation (particularly in the last ten years) is fairly well documented on various sites, with a history of developments in the field.

For this reason, an Internet search, backed up by personal contacts, software analysis and hard copies, was used as the primary search vehicle.

A different method was required for research carried out outside Universities and official channels (schools, associations, support services for the education of blind people) often with only the use of Internet.

On the basis of the information collected in the initial stage of the DDMATH project and the experience of the experts working on the project, it became evident that the computer connected with Braille line and speech synthesis has become the main working tool for blind young people at all levels starting from 10-11 years of age. It follows that users are also interested in specific programs to support mathematics. However, this assumes that two essential conditions for full school integration / inclusion are met with regard to the diffusion of information technologies among young people: adequate training of both supportive and curricular teachers and the presence of accessible and efficient programs (in other words that are inclusive, common for all, or at least open and transparent to sighted schoolmates).

The need and wide use of IT tools (and not only for scientific studies) is equally widespread both in countries where there are schools or special institutes for blind students and in those countries where there is an integrated school within the common one.

The aids are part of an individualized didactic project taking into consideration the type and level of school, whether it is inserted an integrated school or in a special institution. In particular, for the teachers of the integrated school (both curricular and supportive), in the interviews about the needs of users they report of only occasionally finding themselves in the position of having to teach blind children during their career. This is due to the fact that all integrated schools in the area are open to children with disabilities and therefore there are no special schools to which a specific type of users with specific types of disabilities is addressed. All schools take on an inclusive dimension and this requires being able to fill any training needs of the school staff in a short time and to be able to equip the school just as quickly from the request for enrollment of the students. For example, in the case of enrollment of a blind student in a musical high school, it is necessary to be equipped and trained with regard to music in Braille, mathematics in Braille, to be able to quickly know the best reference IT tools to adopt.

In Italy the need to use new IT tools for blind children for the study of mathematics was felt again about thirty years ago, when various experiences were born, especially to allow teachers, even if not very skilled in Braille, to be real supportive to the kids.

Also in Italy, the first innovative and widely adopted solution was the one created by prof. Flavio Fogarolo (teacher and consultant of the CSA of Vicenza) with the Erica program in 1992-96. The Erica program (which will be described in the following pages) is based on the 6-dot braille code and has had the merit of supporting and directing students and teachers to the exclusive use of computers at school with a clear intention of detaching themselves from the use of braille on paper and from the use of typing-braille for secondary school . Erica was not just a program for mathematics, but a real workbook for text, for languages, a vocabulary, a calculator, a squared notebook to do math and finally a school diary.

Erica's experience has allowed the development of the OMNIBOOK program with similar characteristics, but Windows featured.

In those years, some European projects were born in the TIDE sector and third and fourth framework programs with the aim of developing personal computer-based solutions for accessing scientific documents. Among these we remember the projects:

MATH[3] (1997) coordinated by Alistair D N Edwards[4]for the conception of A multimodalinterface for blind mathematicsstudents[5].

TeDUB[6] (2001) coordinated by Geroge Ioannidis[7] A System for Presenting and Exploring Technical Drawings for Blind People.

LAMBDA[8] (2002) coordinated by Giuseppe Nicotra[9] ; Lambda is the acronym for Linear Access to Mathematics for Braille Device and Audio-synthesis.

National projects as the BRAILLEMATH of the Bologna Cavazza Institute (Italy)

Other projects are related to specific products such as graphic embossing printers, as Tiger by Viewplus, or as accessories to other programs such as the mathematical OCR INFTYreader

The LaTeX language deserves a separate chapter, which due to its linear writing and reading method has been widely adopted for mathematical writing for the blind, but as evidenced by many (as for example in the interventions of the recent 13th national seminar in Lisbon on mathematics for the blind of 9 December 2021) turns out to be useful for writing and for being read with special vocal readers, but very complex when used to "do" mathematics and therefore solve complex expressions.

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[3]https://www.cs.york.ac.uk/maths/maths_publications.html (2021)

[4]Alistair D N Edwards - Department of Computer ScienceUniversity of York UK(2021)

[5]https://www.researchgate.net/publication/2814643_A_Multimodal_Interface_for_Blind_Mathematics_Students(2021)

[6]https://cordis.europa.eu/project/id/IST-2001-32366/it

[7]Researcher in Universität Bremen

[8] https://cordis.europa.eu/project/id/IST-2001-37139/it

[9]Arca progetti srl

4.1 Requirements of a new mathematical Braille notations

A number of new Braille notations have been devised for mathematics. It was suggested by Waltraud Schweikhardt (2000)[10] that there are a number of desirable properties for any new mathematical Braille code. They are presented here – with some critical commentary.

R1 The notation must be readable through the finger

Bearing in mind that Braille is essentially read one cell at a time, prefix notations can aid comprehension. Some Braille representations are *not* prefix notations, so that their representation of equation (2) would resemble that in (3).

$$\frac{x+1}{x-1} \qquad (2)$$

$$(x + 1) \div (x - 1) \qquad (3)$$

In other words, reading from left to right it is not apparent to the reader that the expression is a fraction until the division sign (÷) is encountered. A prefix notation (in this case *Latex*, see below) would be

\frac {x+1}{x-1} (4)

R2 The number of characters in a mathematical expression should be as few as possible

This is clearly important in any notation. It is particularly a problem in Braille, given its limitations outlined above, and the fact that multiple cells may have to

[10]Universität Stuttgart: Institut für Visualisierung und InteraktiveSysteme (VIS) See Bibliography

be used to represent a single symbol. The more cells required, the slower and more clumsy the interpretation.

R3 Tactile characters should be understandable intuitively

Visual mathematical symbols often have a graphical interpretation which aids understanding and memorization. For instance, the with the comparative operators < and > the narrower end of the symbol points towards the smaller quantity. Another example would be the use of a superscript arrow to denote a vector (e.g. $\vec{x}$), in which the arrow clearly implies a directional component. The concept can be extended to symmetry, so that symmetry of pairs of symbols, such as parentheses is seen as important.

Although Schweikhardt (op. cit.) advocates this requirement, it is in fact arguable that such associations are as meaningful in the tactual realm, or whether they carry only visual associations. For instance, the symmetry of visual parenthesis is an important cue in facilitating grouping relationships, but that is not necessarily true in Braille notations. Firstly, to recognize the symmetry of tactual symbols requires manipulation of the spatial representation and secondly the serial perception of the Braille does not afford the same immediate grouping that is possible in the visual inspection of a formula.

R4 The notation should be useable in a computer-based learning and working environment

Given the ubiquity of computers, any notation should support computer-based work. They are also increasingly being used to facilitate access to mathematics.

R5 The joint education of blind and sighted people has to be supported

Integrated education is not simply the current fashion; collaboration between blind and sighted people is a necessity. Not the least, it is very common for a blind student to have a sighted teacher. Any notation that they use must be equally accessible. In practice this means that there must be easy two-way translation between any visual and non-visual notations.

Schweikhardt's paper proposes the above 'requirements', but life is rarely that simple. These requirements are not always mutually compatible. Furthermore there are other properties that may be desirable. For instance, compatibility between a mathematical code and conventional literary Braille may be important. More specifically, many people – brought up on six-dot Braille – find the learning and reading of eight-dot codes difficult.

Finally, we believe that it is not enough to architect or have architected a valid 8-dot code if it is not supported by an equally valid software that uses it fully and is more innovative and more complete than a traditional text editor. In fact, if a new code is expressly developed with 8 dots, the reason lies above all in the fact that it can then be used with a personal computer and Braille line and by a program that knows how to exploit all its innovative features. In this regard, the SMSB 8-dot code, which we will analyze later, has not yet been adopted by any program that has been able to convincingly propose it on a national scale, and this is probably the reason for its limited diffusion among users.

4.2 Karlsruhe notation

This notation is based on the 128 characters of the computer ASCII code. By using 8 dots, it is possible to have a simple one-to-one translation from ASCII character to Braille cell. In other words, a formula is translated into a linear notation, that can be represented in ASCII, and then the appropriate cells can be substituted.

So, for instance

$$\frac{x+y}{x-y} \qquad (5)$$

first becomes

$$(x + y) \div (x - y) \qquad (6)$$

which in Braille becomes

This shares the problems of non-prefix notations[11], described above. In this example, it is not until the reader has reached the sixth Braille cell that he or she will be aware that the expression is a fraction (a fact which is immediately apparent to the sighted mathematician).

4.3 SMSB: The StuttgarterMathematikschrift für Blinde

This is the main topic of Schweikhardt's paper (Schweikhardt, 2000) so it is not surprising that it fulfils all the requirements set out therein for a good mathematical code. It is an eight-dot notation and, given the larger set of symbols available, there is a simple one-to-one correspondence between Braille cells and mathematical symbols (including separate representations for letters and numbers). This facilitates translation between visual (but linearized) print notation, visual Braille and tactual Braille.

It is the first complete 8-dot code that has been made.

It was a very innovative code because it was designed to be used on a personal computer. it is studied, in its structural setting, in an analogous way to the notation of the sighted person. MSM follows many features of Marburg notation.

Unfortunately, it did not have a practical use and a mathematical writing program was not developed on this code.

The teacher, W. Schweikhardt, has carried out numerous researches in the field of mathematics coding for the blind, and there are numerous publications of hers for the search for an optimal representation code.

11 In this case an 'infix' notation.

An in-depth description of the SMSB code is available in this Article: REQUIREMENTS ON A MATHEMATICAL NOTATION FOR THE BLIND Waltraud Schweikhardt 1 downloadable from the following site:

https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.33.7614&rep=rep1&type=pdf

A list of her publications is available on this web page:

https://www.semanticscholar.org/author/W.-Schweikhardt/2155362

4.4 Latex

Tex(Knuth, 1987) was developed as a typesetting (markup) code, capable of representing a wide range of printed notations – including mathematics. It was not intended as a notation for blind users. Subsequently Latex (Lamport, 1988) was developed, based on Tex, and designed to be more usable. (Its name is derived from 'Tex for Laymen'). Latex has been widely adopted by those who write technical texts. Latex is very much oriented towards the needs of printers and not of mathematicians, since it was originally intended not as a notation to be read in its raw form by human readers, but to be first interpreted by typesetting software before being read by people.[12]

Nevertheless, Latex has been used widely by blind mathematicians. There are a number of reasons for this. Firstly, it is very expressive, most printed mathematical notations can be expressed in Latex. Secondly, it is used very widely by sighted mathematicians as a means of publishing their work. It thus can be used equally by blind and sighted mathematicians. The sighted ones can view it as rendered by a typesetting program while the blind ones can read the raw markup – usually using a computer with a screen reader which renders the text in synthetic speech or Braille.

Latex is a prefix notation. It is quite verbose in the sense that it uses words to represent symbols. We have already seen the example (4) above, where a fraction is signalled by the word '\frac'.

[12] Addition information: http://www.integr-abile.unito.it/backup/latex.php (last access 2021)

From the comments that emerged from the interviews with users, it was found that LaTeX is rarely used in Italy and France, while it is better known in Germany and Poland. It follows that support teachers of the integrated school such as the one present in Italy, (and this consideration could be expanded for similar situations in Portugal, partly in Spain and France), are not so inclined to learn and adopt a computer marking language for the graphic description of a page, considering this solution too complex and of little use for the common blind children of the first or second grade secondary school. LaTeX, on the other hand, could be a good solution for blind students who choose university science courses and who are particularly gifted with good working memory, logic and abstraction skills.

The interest in LaTex and its possible use for blind children, however, has not waned over the years, and at university level there are numerous studies, publications, degree and doctoral theses, researches to expand its use, and exploit linear representation and reading through speech synthesis. Here are some LaTeX-based proposals.

4.5 Latex-to-speech translators: Aster and MathTalk

Programs have been written which will render Latex in a spoken form. Aster (Raman, 1994) is a reader for Latex in general, including mathematics, while MathTalk[13](Stevens and Edwards, 1994; Stevens and Edwards, 1994; Stevens, 1996) was specifically designed to render the mathematical Latex. Note that many of the ideas developed in MathTalk were later incorporated in the Maths Workstation (Edwards, 2003).

Aster[14] (Audio System for Technical Readings) is described as ‘a computing system for speaking technical documents’ (Raman, 1997), p. 8). In other

[13] Robert Stevens, Alistair Edwards, Mathtalk: The design of an interface for reading algebra using speech, Conference paper, First Online: 08 June 2005, Part of the Lecture Notes in Computer Science book series (LNCS, volume 860)

[14] T.V. Raman, Audio System for Technical Readings (Lecture Notes in Computer Science, 1410) 1998th Edition

words, it is intended to render any document marked up in Latex in a readable form – including (but not limited to) mathematical portions.

Mathematical notations are of necessity precise. For instance, the meaning of the following two equations is completely different even though they differ visually by just two symbols:

$$3x + 4 = 7 \tag{7}$$

$$3(x + 4) = 7 \tag{8}$$

A problem with speech renderings of mathematics is to retain the required level of unambiguity is hard to achieve. Both Aster and MathTalk use a technique of verbal encoding to mark the grouping of subexpressions, so that (7) would be rendered by MathTalk as

'Three x plus four equals seven.'

Whereas (8) would be

'Three times the quantity x plus four, end quantity, equals seven.

A systematic scheme for such verbal renderings of mathematics was devised by (Chang, 1983). However, MathTalk also had a facility whereby the syntactical groupings could be signalled using the prosody15 of speech. A mathematician, such as a lecturer, used to speaking mathematics, can signal the groupings by varying qualities such as timing, pitch and rhythm. For instance, (8) would be spoken as

'Three, x plus four equals seven.'

with a pause after the word 'three' and the bracketed subexpression ('x plus four') being spoken at a slightly lower pitch.

[15] 'Prosody refer collectively to the qualities of stress, rhythm, timing and intonation' (Edwards, A. D. N. (1991). *Speech Synthesis: Technology for Disabled People*. London, Paul Chapman., p. 17)

Aster and MathTalk have had the merit of having initiated and deepened the various existing problems for reading mathematics, and today their works are part of the literature on this subject.

4.6 A LaTeX to Braille Conversion Tool for Creating Accessible Schoolbooks in Austria: Labradoor

Labradoor is a program that translate Latex into Braille. According to the LaBraDoor (LaTeX to Braille Door) system, a tool that allows textbooks to be automatically converted from their TeX source into an accessible equivalent in Braille. The revamped version makes use of state-of-the-art technologies and techniques for automatic Braille document generation, including math and other non-linear information widely used in educational materials. (Batusic, Miesenberger et al., 2016).

4.7 ASCII-Mathematikschrift (AMS)

Most computer software uses an encoding of symbols known as ASCII. ASCII is the American Standard Code for Information Interchange (although it is also an ISO standard). There are 127 codes defined of which 95 represent visible characters: upper- and lowercase letters, digits and punctuation[16]. (See figure 5). This character set can provide the basis for another approach to an accessible mathematical notation. A code which represents all the required mathematical symbols using only the characters in the ASCII set has the advantage that any mathematics can be represented in this widely available code. This was of particular value in the time when printer technology was more limited and printers could only produce the characters in the ASCII set. At the same time, if you have a Braille code which encompasses all the ASCII characters, then you can produce arbitrary mathematical expressions in Braille. Of course, given that there are 95 ASCII characters, 6-dot Braille is not

[16]The remaining 32 codes are used mostly as a means of controlling devices. For instance, different codes will cause a printer to move to the next line, to the end of the page and such-like.

sufficient to represent this set (restricted though it is), but 8-dot Braille codes can.

space	!	"	#	$	%	&	'	(	)	*	+	,	-	.	/
0	1	2	3	4	5	6	7	8	9	:	;	<	=	>	?
@	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
P	Q	R	S	T	U	V	W	X	Y	Z	[	\	]	^	_
`	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
p	q	r	s	t	u	v	w	x	y	z	{	|	}	~	

Figure 6: The printable characters of the ASCII character set.

It is beyond the scope of this document to go into a complete description of the code, but instead some examples of AMS[17] are given in figure 6. In the following description the number of entries in figure 6, which illustrate the given point, are given in parentheses.

There are a number of conventions used to translate symbols. Spaces *are* significant. (6, 9). The apostrophe is used to change the meaning of the following symbol. Thus, a percent sign preceded by an apostrophe represents the infinity symbol (1). Greek letters are designated by the Latin letter preceded by a question mark (2). Some operators are described by function names (3, 6). Interestingly, some of the translations seem to have been chosen because of the visual similarity between the ASCII representation and the conventional notation (10). Whether this aids reading in the Braille is questionable. It may be, though, that an explanation of the choice of symbols does aid recall.

Example	AMS	Conventionalnotation
1	'%	∞
2	**?a**	α
3	**Sum[k=1;n]**	$\sum_{k=1}^{n}$
4	**x**2**	x^2

[17] https://chezdom.net/wp-content/uploads/2008/11/ams.pdf (2021)

5	`x(i;j)`	x_i^j
6	`Al x`	$\forall x$
7	`(a+b)/(a-b)`	$\frac{a+b}{a-b}$
8	`lim[x->?x;]`	$\lim\limits_{x \to \xi}$
9	`sin x`	sin(x)
10	`_|_`	$\perp$

Figure 7: Examples of AMS and corresponding conventional notation.

4.8 Lambda code

LAMBDA project IST-2001-37139 (Linear Access to Mathematics for Braille Device and Audio-synthesis) is a Maths writing system for computer improved to be used with Braille display and voice synthesis. It is developed within a IST three-year long European project to which 15 partners from 8 different countries take part.

LAMBDA is based on a linear writing system: maths formulas are written textually with a regular sequence of same-sized characters.

The system forecasts the full compatibility with the maths marking language MathML version 2.0, defined by the international Consortium W3C, which represents today the conversion and transformation standard widely spread all over the world. Through the MathML the LAMBDA code is convertible, in input and output, with the best known maths writing formats.

Through the MathML it is possible to obtain straightaway the graphic version of the mathematical text, to be destinated to either printing or visualization on-screen.

The Lambda project realized 2 important parts:[18]

[18] https://cordis.europa.eu/project/id/IST-2001-37139/results/it

The 8-dot mathematical Lambda code –

The mathematical Lambda editor

The first one consists of a linear mathematical syntax and an 8-dot representation adapted to the 6-dot Braille combinations, traditionally used in the various countries considered.

The second one is a software product consisting of a writing editor and a didactical help for the solution of mathematical expressions in 8-dot Braille.

Since the code is connected to an editor with a didactic approach, the whole becomes a system, and being one of the most widespread at European level, a complete chapter will be dedicated to it in the following pages.

4.9 Gardner–Salinas braille code

The Gardner – Salinas braille code (or GS code) was developed as a unified braille code to be used not only for mathematics. Unfortunately, this philosophy was rejected by the Unified English Braille Code Development Committee, and therefore did not have further future.

Ulteriori informazioni sono disponibili sulla pagina web:

https://en.wikipedia.org/wiki/Gardner%E2%80%93Salinas_braille_codes

5 Readers, speech syntheses for LaTeX documents

5.1 The Axessibility package by Laboratory Polin, University of Turin

❖ **General remarks**

The Project "Axessibility" of the Polin laboratory of the University of Turin[19] concerns the theme of reading through vocal synthesis of the mathematical texts of PDF documents

The project page (last access 2021) reads as follows[20]: *PDF documents containing formulas generated by LATEX are usually not accessible by assistive technologies for visually impaired people and people with special educational needs (i.e., by screen readers and braille displays). The Axessibility package manages this issue, allowing to create a PDF document where the formulas are read by these assistive technologies, since it automatically generates hidden comments in the PDF document (by means of the /ActualText attribute and/or suitable tags) in correspondence to each formula*

The aim of the project is therefore to develop and test new accessible and inclusive technologies for the study of STEM by people with visual and sensory disabilities.

A video presentation of "Axessibility" is available on the DDMATH project website:

https://ddmath.eu/cosa-e-il-latex/

[19]https://ctan.mirror.garr.it/mirrors/ctan/macros/latex/contrib/axessibility/axessibility.pdf (last access 2021) Lab presentation: http://www.integr-abile.unito.it/documenti/accessibilita_materiale_didattico_labo.pdf(last access 2021)

[20]https://ctan.org/pkg/axessibility (last access 13-09-2021) script: https://github.com/integr-abile/axessdicts(last access 2021)

❖ Innovative elements

The innovative aspect is undoubtedly that of making PDF documents with scientific and educational content, written in LaTeX, accessible to people with visual disabilities. In fact, it consists of a LaTeX package that allows to create PDF documents in which mathematical formulas can be read by means of a screen reader and Braille bar, and likewise on the screen in LaTeX format.

The whole is produced automatically by generating specific comments in PDF documents (with the / ActualText attribute) in correspondence to the formulas, and they contain the LaTeX code of the corresponding formulas.

❖ Limitations and problems

The limit is linked to the issue of the use of LaTeX as described above, which imitates its use in the university environment and is in any case dedicated to students with particular skills and abilities. Likewise, a specific competence is required of support teachers, personal assistants of integrated schools. A different thing happens at special institutions, for example in Germany or Poland, where LaTeX is introduced already in the high school by specialized teachers and for students with strong scholastic skills.

❖ Aspects which may be useful for the teacher

Although LaTeX was not born as a tool for accessing mathematics for blind children, it is increasingly establishing itself among students who today more than ever enroll in university science courses. To write formula in LaTX you just need a simple text editor and then transform it into an accessible PDF using the “axessibility” software.

If the teachers of the school should want to approach LaTeX and the axessibility package, they can refer to the Polin Center of the University of Turin which, thanks to its large working group, promotes various training activities and numerous publications on their project.

5.2 Aster

T.V. Raman - Cornell University[21]

❖ General remarks

Aster (Audio system for technical readings) is a program created in 94 by T.V.Raman, a blind student and Ph.D. at Cornell University in Ithaca - New York, USA.

The aim is to obtain a clear reading by voice synthesizer of mathematics texts in TeX .

Raman is interested particularly in the rules of spoken mathematical communication.

❖ Innovative elements

Raman tackles the problem of how to transfer all the information in a mathematical text clearly and efficiently – and above all the structure of the information – by voice, using the full potential of the electronic document to separate the rendering from the content.

He writes:

AsTeR is founded on the belief that all information is display-independent. Information has structure, and this structure is rendered on paper or on a visual display, but the information itself is not restricted to these output modes. Thus, AsTeR renders this same information in audio. AsTeR recognizes the logical structure of a document as embodied in the markup source and represents this structure internally.

http://www.rit.edu/~easi/itd/itdv01n4/article2.html

Aster can produce different renderings of the same document, according to user preferences, the topic, the need to include detail and other factors. The user can also define new rules for customized rendering.

[21] https://www.cs.cornell.edu/info/people/raman/current/aster/aster-toplevel.html (last visit: 2021)

❖ Limitations and problems

In addition to the technical problems of the sw and hw technology used in 1994, the principal limitation of the system is the fact that Aster is a reading instrument and hence cannot be used for the direct interactive management of mathematical text.

Access by Braille and not also through tactile Braille is also a limitation.

Finally, as mentioned above, **the program has not been further developed.**

5.3 TESI

Massimilaino Zattera – Giuliano Artico University of Padova – Italy

TESI is an acronym of «TeX to Speech Interface». It was the University thesis of Massimiliano Zattera when graduating in Engineering at Padua University in 1997, under the supervision of the blind Mathematics Professor Giuliano Artico, of the Department of Pure and Applied Mathematics at that University.

The aim of the project was to develop an efficient system for the consultation of mathematics texts in TeX format, with voice syntheziser, suitable above of for blind University students.

The prototype produced by the thesis research has not been further developed

5.4 MATHS

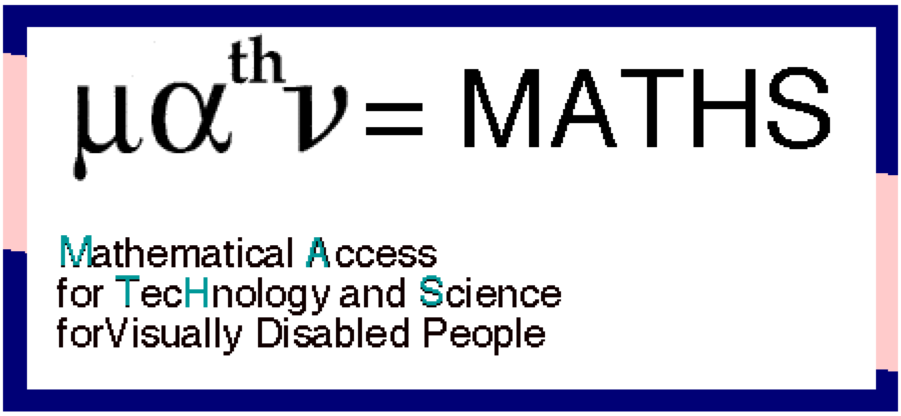

Figure 8: Logo Math project

Maths (Mathematical Access for TecHnology and Scienze for Visually Disabled People), begun in 1994 and terminated in 1997, is a complete project aiming to create a multimedia work station for the blind or visually disabled student. Based on SGML (Standard Generalized Markup Language), it does not seek to create a machine for teaching mathematics but to supply “pen and paper”. The role of the teacher remains central for the learning of specific roles in the discipline.

The Maths Project (Mathematical Access for TecHnology and Science for visually disabled people) is a European project that precedes the Lambda Project. It was funded under the Tide Initiative (TP1033).

The project’s objectives were to develop a multimodal workstation that would make mathematical notations accessible to visually disabled students.

The MATHS project is mentioned as it had an important impact in the years starting from the end of the 1990s. Unfortunately, it was not developed further, but it did allow researchers of the time to properly target and further develop activities. Among the projects that have been able to grasp the positive elements of the Maths project, the LAMBDA project is cited as an example.

5.5 MathTalk

MathTalk[22] is a product developed in Texas which (unlike the programs seen above that read mathematical texts such as LaTeX) allows to write mathematics under dictation. In fact, it is a mathematical software for speech recognition MathTalk that relies on a better known software, the Dragon® NaturallySpeaking. The program is completed with Scientific Notebook which allows users to translate mathematics in Braille in Duxbury Braille Translator or other mathematical publishing programs such as Lambda. Unfortunately, MathTalk was withdrawn from the market in 2021 and **will no longer be developed.**

[22] https://mathtalk.com/ (2021)

The EUROmatH Project

During the last few years (2017-2019) Poland, Ireland and the Netherlands made a significant effort to facilitate E-learning of mathematics by visually impaired students.

Due to the cooperation between these three countries the EUROmatH Project was carried out. http://www.euromath.eu

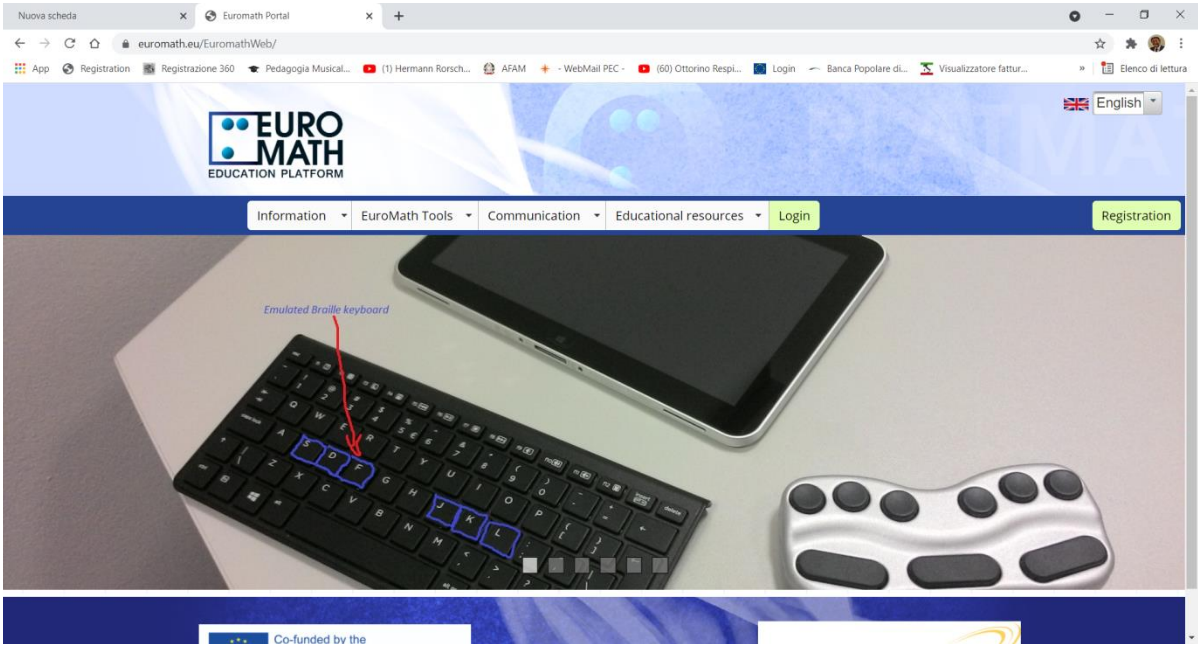

Figure 9: Web site Euromath portal

The main partners of the project were:

Research Institute of the Polish National Academic Network (Poland), Dublin City University (Ireland) and VISIO (The Netherlands).

The aim was to create an educational platform in the field of mathematics, designed both for sighted teachers and visually impaired students from primary and secondary schools.

The platform consists of two parts:

- Web application meant to teach visually impaired students from primary and secondary schools by sighted teachers,

- Euromath portal containing educational resources accessible for students in English and Polish.

The application enables a student to edit mathematical documents (including formulas, graphics etc.) and navigate through them. All this can be done in Braille and/or speech. The teacher can see the content manipulated by a student in a graphic form. Another valuable feature is that a visually impaired person can “view” mathematical graphics on the touch screen in the form of sound signals and spoken description. The student can also create the graphics and insert it in a document. Such a mathematical document can be saved as a file in the epub3 format.

So, the EuroMath online application for teachers and students allows to create content and solve tasks.

The two additional items offering support are:

- ✓ Windows EuroMath CUBARITHM application for students to learn arithmetic calculations by writing (below the line)
- ✓ CALCULATOR application for blind students

The portal serves the teachers and students to store and exchange the mathematical materials. You can download the EuroMath software and mathematical content from it.

The materials can be prepared in the form of worksheet, quiz and task, such as “join in pairs”, “multiple choice” and many others.

In the framework of the project teachers from the three countries created up to 300 units.

To write formulas the Polish student can use the Polish Braille Mathematical notation (slightly modified Marburg Notation) and the English speaking one – Unified Braille Notation.

In addition, it is possible to write using ASCII Math Notation.

The teachers do not have to design materials especially for their visually impaired students, because all the mathematical texts are accessible both for the blind and for the sighted.

6 Mathematical software and Braille editor

6.1 Erica

Flavio Fogarolo - Vicenza Education Department– Italy

The Erica program (www.provvstudi.vi.it/erica) is a project over a number of years which was initiated in 1993 by "New Integration Technology", part of the Vicenza Education Department (a local branch of the Education Ministry).

The potential of IT for blind users was evident back then, but there were a number of concrete barriers to overcome, for example the lack of IT skills by teachers and the excessive complexity of the management programs, not at all suited to use by children.

The Erica program tackles the problem with a simple, immediate environment, suitable for even small children, and is particularly useful for teaching needs.

The problem of the access to mathematics was faced in 1994 with a system based on a linear code and some input and manipulating functions by the editor

The program was disseminated in 1995 by the Italian Blind Union; the handbook with guidelines for teachers was printed in 1995 by the Italian Library for the Blind. The product was then made available on Internet.

Erica was the most used program by the blind in Italy. Currently it is no longer developed, but it is reported in this work as it was the inspiring program of the Lambda program.

6.2 WINTriangle

John Gardner University of Oregon, Norberto Salinas University of Kansas

Wintrianglewas created by the Oregon State University Science Access Project, aimed at creating an instrument for students of science and mathematics with special attention to the access of blind or visually disabled students.

The program no longer appears to be in distribution and is mentioned here for completeness as it was a program that had a wide circulation having also had the support of ViewPlus Technologies, Inc.

In addition to the mathematical word processor, Triangle includes a calculator of expressions, the display of graphs and tables, and a program (Touch–and Tell) for listening to, and/or reading assisted by Braille, tactile figures on an external pad.

Basically it is a mathematical editor manageable by the blind, associated with a linear GS mathematical code.

More information can be found in a 2016 article by John Gardner[23]

6.3 ViewPlus

❖ General remarks

Figure 10: ViewPlus braille printer

ViewPlus is an American company from Oregon founded by prof. John Gardner who makes graphic embossing printers. This review is inserted as it allows the relief printing of drawings, such as graphs, tables, and other mathematical documents.

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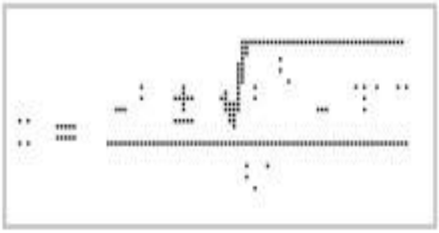
[23]Accessibility, Tactile Graphics emerging Computer Technologies for Accessible Math, Posted On August 25, 2016 By John Gardner, https://viewplus.com/emerging-computer-technologies-for-accessible-math/ (last access 10/06/2021)

DotsPlus® as it appears visually on the computer screen and on a printed page.

DotsPlus® as it appears in Tiger Viewer to be embossed on the Tiger Embosser.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

⇔



❖ Innovative elements

The entire ViewPlus system is innovative, particularly the inclusion of tactile graphics, with the form of conventional mathematical symbols, input next to the Braille font with a similar spatial lay-out.

❖ Limitations and problems

The main drawback of ViewPlus is that it is only a system for reading math: a blind student cannot compose and manipulate formulae and graphics. Only a sighted operator can write math text with this method.

To write math a student needs to use not only another instrument, but also a different code: the need to learn two different codes (one to read and the other to write math) is a big drawback, didactically speaking.

❖ Aspects which may be useful for the teacher

Mathematics is made up of symbols, but also of graphs, and besides there is the whole aspect related to geometry that is still not very accessible for the blind; moreover the graphic representation is complex to solve. A graphic printer can be a solution and a valid alternative to other tools such as the rubber surface or the microcapsule oven. A printer allows, with good reproduction precision, speed of execution, the easy learning of flat geometric shapes and various objects, the representation of functions through graphs, also useful for chemical formulas

6.4 MAVIS

Mathematics Accessible To Visually Impaired Students

❖ **General remarks of the product**

Mavis is a tool for an easy communication between math teachers and Blind students. It is possible to read in web site:

https://www.dinf.ne.jp/doc/english/Us_Eu/conf/csun_99/session0255.html (last access 2021)

There are currently two problems blocking this communication:

- ✓ Teachers cannot easily make their mathematics and science materials accessible to these students.
- ✓ Blind students cannot independently communicate their course work in print.

In solving these two problems, we expect to give visually impaired students better access to the fields of science, engineering, mathematics, and technology. Thus, the objectives of the MAVIS project are to:

- ✓ Create tools and channels that will allow mathematics instructors nationwide to give their visually impaired students adequate materials,
- ✓ Create tools that will allow visually impaired students to produce standard-quality print technical documents for their instructors,
- ✓ Ensure that in-class electronic devices are accessible,
- ✓ Ensure that all of these innovations are practical, user-friendly, and intuitive to learn.

We are committed to research and are also a production lab for Braille and tactile images.

The research is focused on:

- ✓ development of print math to Braille (Nemeth code) conversion software;

- ✓ development of an audio screen reader/browser for mathematics;
- ✓ development of Braille math (Nemeth code) to print conversion software;
- ✓ testing of presently available access technology.

MAVIS provides support for NMSU students who need classroom materials prepared in alternative media. Following the NMSU guidelines, MAVIS meets with the student and professor prior to the start of classes to help outline the student's needs and individual production procedures. For interactive classes, MAVIS also provides an in-class scribe/reader who prepares Braille or tactile drawings of visual presentations.

The conversion software of working group MAVIS is the Scientific Notebook to Nemeth Code Converter© (1998)

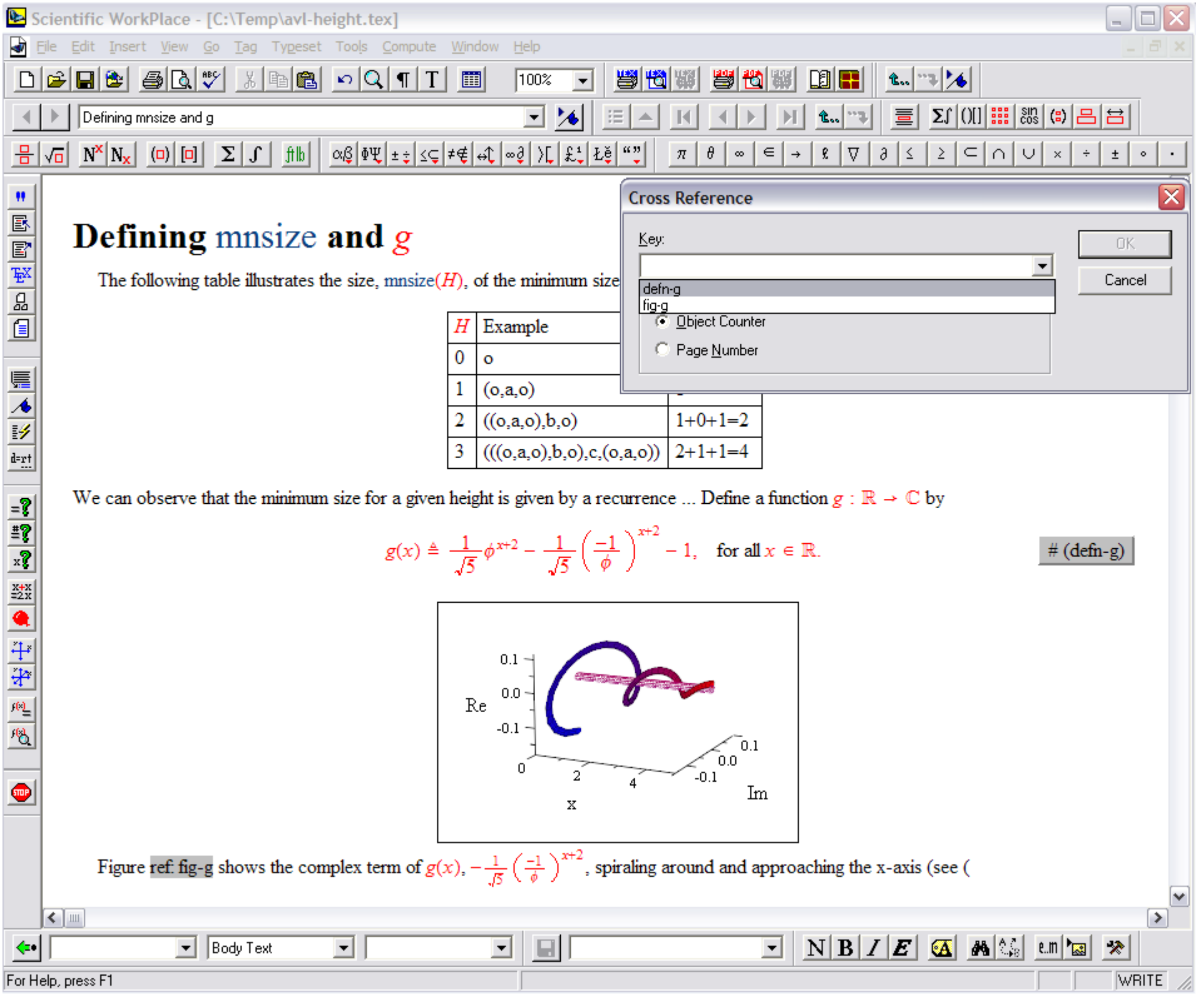


Figure 11: Scientific WorkPlace (source https://en.wikipedia.org/wiki/Scientific_WorkPlace)

The Scientific Notebook to Nemeth Code Converter is a mathematical word processor allowing graphics to be included in documents. The documents created are saved in Tex (Latex) format and are ready for conversion into Nemeth code by means of a special translation filter. The site includes the following description:

"The Scientific Notebook to Nemeth Code Converter© is the latest in Nemeth code generation technology. Scientific Notebook by MacKichan Software Inc., is best described as a mathematics word processor. This software is used in the creation of mathematical documents, allowing seamless incorporation of mathematics, text, and graphics. Documents created with Scientific Notebook may be saved as LaTex (.tex) files and are then ready for use with the Nemeth Code Translation Filter©. In addition, Scientific Notebook is powered by the Maple V computation engine, which allows the user to do mathematical computations automatically. You can also read it with a refreshable Braille display if you wish.

Unfortunately, MacKichan Software, Inc. ceased operation on 30 June 2021 and sales have ceased for Scientific WorkPlace, Scientific Word, and Scientific Notebook

The latest released version of these products is 6.1.2.

References: https://www.mackichan.com/

❖ Innovative elements

The strong point of MAVIS is the Scientific Notebook, a mathematics word processor which can be used simply and easily. LaTeX could only be used by experts and in this format in which documents in the Scientific Notebook are saved.

The Scientific Notebook converter to the Nemeth Code also makes transcription into Braille unnecessary, allowing teachers to get documents ready and print them out in short time.

The technology for formal language processing is the heart of the innovation in this system. Latex, the language used for presentation, is converted into

Nemeth, the language used for the content, with a precise contextualization each translation step. In some cases Braille rules for the text are changed to prevent ambiguities.

❖ Limitations and problems

The main limitation of MAVIS is that blind students cannot produce mathematical documents independently. No conversion yet exists from Nemeth to any format which can be printed out.

The official site says:

"*One problem that has persisted for mathematics teachers of blind students is that blind students cannot independently create print mathematics. Consequently, either their teachers have had to learn Braille, or student and teacher have had to agree upon a new code for the submission of classwork. MAVIS is using language processing techniques to create a tool with which blind students can write mathematics in Nemeth Code Braille and then have it automatically transcribed to print. This would save the student from re-doing completed work, and it would also save the teacher from having to learn complicated alternative mathematics notations*."

The Scientific Notebook does not support MathML language.

It is not accessible through voice software.

6.5 LABRADOOR - LaTeX-to-Braille-Door

University of Linz

❖ General description

Labradoor is a program which prepares scientific text composed in LateX for printing in Braille, creating greater access to blind students. Created from a broader research project at the University of Linz dealing with problems of access to Mathematics, Labradoor is used to produce mathematical texts for students at secondary school and University.

Additional information:

https://www.researchgate.net/publication/304822083_A_LaTeX_to_Braille_Conversion_Tool_for_Creating_Accessible_Schoolbooks_in_Austria (2021)

The general features of the program are

"LABRADOOR is planned and developed as a module of a computer supported mathematical working environment for the blind. It gives access to mathematical sources by representing documents in a format which suits the needs of

- ✓ the mathematical working environment
- ✓ the computer supported production of Braille outputs and
- ✓ the production of electronic documents.

LABRADOOR acts as a LaTeX-to-Braille-door. LaTeX is a system to produce and to layout texts with mathematical formulae in excellent quality. The program got extremely widely used for the production of mathematical and scientific publications. Therefore, especially in our practical support at university, it is necessary to open the blind a door to LaTeX.

Although LaTeX can be read directly by the blind it does not fit their needs because of the huge amount of layout information and the lack of ergonomic in Braille. There are a lot of different notations available which better fit the needs of the readers of mathematical formulae. In the German speaking area, the "InternationaleMathematikschrift für Blinde" also called "Marburg Notation", is a notation which, although by far not universally used, has gained some popularity among educational institutions for the blind. It can be seen as a suitable and accepted standard for reading mathematics. Students ask for materials in that format which they learned to use. As a first step for the German speaking area, the system translates LaTeX into grade one Braille and thereby the mathematical parts of LaTeX into Marburger Notation.

The documents handled by LABRADOOR have to fit the LaTeX conventions concerning the organisation of input data and mark-up language. Besides the elementary input conventions described in the "LaTeX Book", LABRADOOR supports some packages changing and/or extending input conventions:

Packages `German' and nearly all options of the package `inputenc', which allows one to use different code pages are supported.

All literals, accented literals (being special LaTeX commands), punctuation marks and numerals are mapped according to the output table (the currently chosen code page for the Braille printer or for the Braille display) and according to the active language.

Users are free to implement the support for other languages supplying LABRADOOR input files for hyphenation and 8 dot Braille and 6 dot Braille tables suiting these languages.

The user extendible implementation of different shorthand systems is planned.

If LABRADOOR encounters a formula within a document, it enters the corresponding mathematical mode. There are currently three implemented maths modes:

- ✓ in-line mathematics mode (the formula is placed in the text flow of the current paragraph),
- ✓ displayed mathematics mode (the formula is placed in its own line of text), and
- ✓ equation mode (just like displayed mode, but with an additional automatic numbering of equations

Layout features:

LABRADOOR can interpret the layout description of LaTeX for the production of the layout of Braille material.

http://www.snv.jussieu.fr/inova/publi/ntevh/labradoor.htm (2021)

❖ Innovative elements

The Labradoor project represented is one of the first and most important attempts to automize the transformation of mathematical text form normal standard formats to print Braille.

Starting from linear mathematical notation, such as TeX and LaTeX, the structure code is very similar to Braille so it is possible to, within certain limits, to automatically transform mathematic text normally used by the blind in tactile writing.

Paolo Graziani[24] writes:

"*This approach has the advantage of using a common code for printing both hard copy and Braille, although there are some limitations for the latter. It is not necessary in this case for the transcriber to know the details of the Braille mathematics notation, because the TeX code requires an analysis of the mathematical symbology to be used in print expressions and formulae, in the hard copy as well, with a sequencing of various parts and respect for the language syntax, to obtain the required graphic result. This code can be interpreted in an alternative form to obtain the corresponding Braille version automatically, so that is enough for the transcriber to know the interpretation limits for the Braille print and follow certain rules concerning the use of language, when the text is destined for Braille.*"

Translation from the web site: www.handimatica.it/Atti98/C98grzia.htm (2021)

❖ Limitations and problems

The only Braille mathematical implementation available is the Marburg notation; there are no conversions with other national codes.

The autonomous use of the Labradoorsystem by blind students is difficult to achieve because inputs and outputs are based on different codes: the mathematics text must be input in TeX code and print output follows Braille rules.

Graziani comments:

"*It can be said that TeX, although an efficient language for printing mathematics texts for the blind, as an entirely textual descriptive language and hence manageable by any editor, also in practice fails to meet this aim. In principle the method could be used to write and print classwork. But, in practice, this is practically impossible because of the difficulties in rereading*

[24]Paolo Graziani, recently deceased, he was a researcher of the National Research Council, "Nello Carrara" Institute of Research on Electromagnetic Waves

the text coded in TeX, a language more oriented to the machine than to man. It would be unthinkable to carry out classic mathematical operations on expressions written in TeX.

This way of producing mathematical texts may interest a blind student to produce books, articles, written reports or OHTs, where no interaction is required to schedules with complex expressions and equations, but may be prepared beforehand with another system and translated into TeX only for print reasons with ordinary hard copy notation."

❖ **Aspects which may be useful for teachers.**

Pioneering work such as this gives a much clearer picture of the problems involved in the transformation of a linear text for graphic display (such as LaTeX) in the equivalent, still linear, version, destined for tactile reading.

Much more than TeX, MathML has the role of exchange code, is standardized and shared internationally.

XML, which is at the base of MathML, allows a system of functional and flexible notation to be designed, able to interact with all national Braille codes.

6.6 BraMaNet

Handicap section of the Lyon 1 University (France)

❖ **General information**

The BraMaNet software was developed by the Handicap section of the Lyon 1 University (France) as part of a university thesis. It is a freeware under license GNU/GPL (this license guarantees freedom to copy, to modify and distribute it).

BraMaNet is an XSL Style Sheet that translates MathML (Presentation tags only) into French Mathematical Braille. It has a user-friendly VB interface and can be used together with MathType to translate Word Documents into Braille for printing.

The implemented symbols and expressions can be used at the present time until the last year of high school.

- ✓ The mathematical symbols Braille table can be adapted to any Braille device (ephemeral Braille device or Braille printer).
- ✓ The transcribed file is a standard textual file so it can be easily used by any other software of Braille transcription like Duxbury or Braillestar.
- ✓ BraMaNet can provide documents with the two mathematical standards. However the new version will use only the new mathematical standard of September 2001.
- ✓ There are short cuts keyboard for all the current actions.

The last version is the version 2.2.5c of the 4th March of 2003.

❖ Innovation elements

A Braille transcription software for mathematics did not exist in France. From this point of view, BraMaNet is an innovation.

BraMaNet transcribes mathematical formulas written in the MathML language. This format (belonging to the recommendations of the W3C since April 1998), is recognized by all major navigators and allows to introduce mathematics on the Web.

With the script " MetaBraMaNet " the Braille transcription of equations can be done directly from a Microsoft Word document (formulas being written using MathType).

This script is composed of two translation steps:

- ✓ the first step uses MathType to translate the Word document into MathML.
- ✓ the second step uses BraMaNet to convert MathML language into Braille.

❖ Limitations and problems

Because of no coding of the mathematical " meaning ", some choices of translation had to be made. BraMaNet treats only the presentation aspect of the MathML language.

A file called "Limits of BraMaNet" illustrates the problems which follow. It can be downloaded from the following address:

Link no longer active: http://handy.univ-lyon1.fr/projets/bramanet/tests /

A- Choices carried out

- ✓ Dots 6 and 3 (beginning of formula) are put before each mathematical expression. Isolated letters or numbers and measuring units will be preceded by dot 6 or no sign according to the representation of the letter (capital or small) and if the document is in grade 2 or grade 1 Braille. In spite of that there remains some litigious cases.
- ✓ For " complex " superscripts, indices and roots, either the sign of end (dots 23) is placed or there is constitution of blocks.
- ✓ For symbols which are at the same time superscribed and subscribed (combinations, sums...), superscript (written after the subscript) will be always considered as complex to avoid ambiguities. It is the same for integrals.
- ✓ In the case of integrals, the integrated function is theoretically considered as a block but it is very difficult to delimit it. We consider that the beginning is after the superscript and the end after " dx " or any other integrated variable.
- ✓ For the fractions, if the numerator and/or the denominator are " complex " it will be placed between brackets or there is constitution of blocks.
- ✓ The exclamation mark is considered as meaning " factorial " except if it is preceded by the symbol " it exists ". It means then " unique ".

B - Limits and problems of transcription

The main transcription errors using BraMaNet are due to the fact that the same sign used by blind persons can be transcribed into Braille in different ways according to its mathematical significance. However, as it was said previously, starting from an equation written under Word, it is not always possible to recognize the mathematical meaning of a sign.

Some problems were already identified during the development of BraMaNet:

- ✓ Recognition of measuring units: they are identified as being letters preceded by spaces and numbers. " 2 km " will be thus recognized as such but not " 2km " which will be considered as a formula. It is the same for the complex units like " 20 km/h " or " 2 m^2 ".
- ✓ For trigonometrical symbols, logarithms…, it is not possible to know where the argument finishes. It is possible to get over this disadvantage with brackets placed during the input, e.g. cos(2x).
- ✓ Every copy/paste towards MathType is interpreted like a text document and not like a mathematical expression.
- ✓ BraMaNet cut the long expressions according to the desired length of Braille lines but cannot take into account the characters preceding the expression. The total length of a Braille line can be sometimes longer than planned. To resolve that, the long expressions are placed at the beginning of a line.

6.7 NatBraille

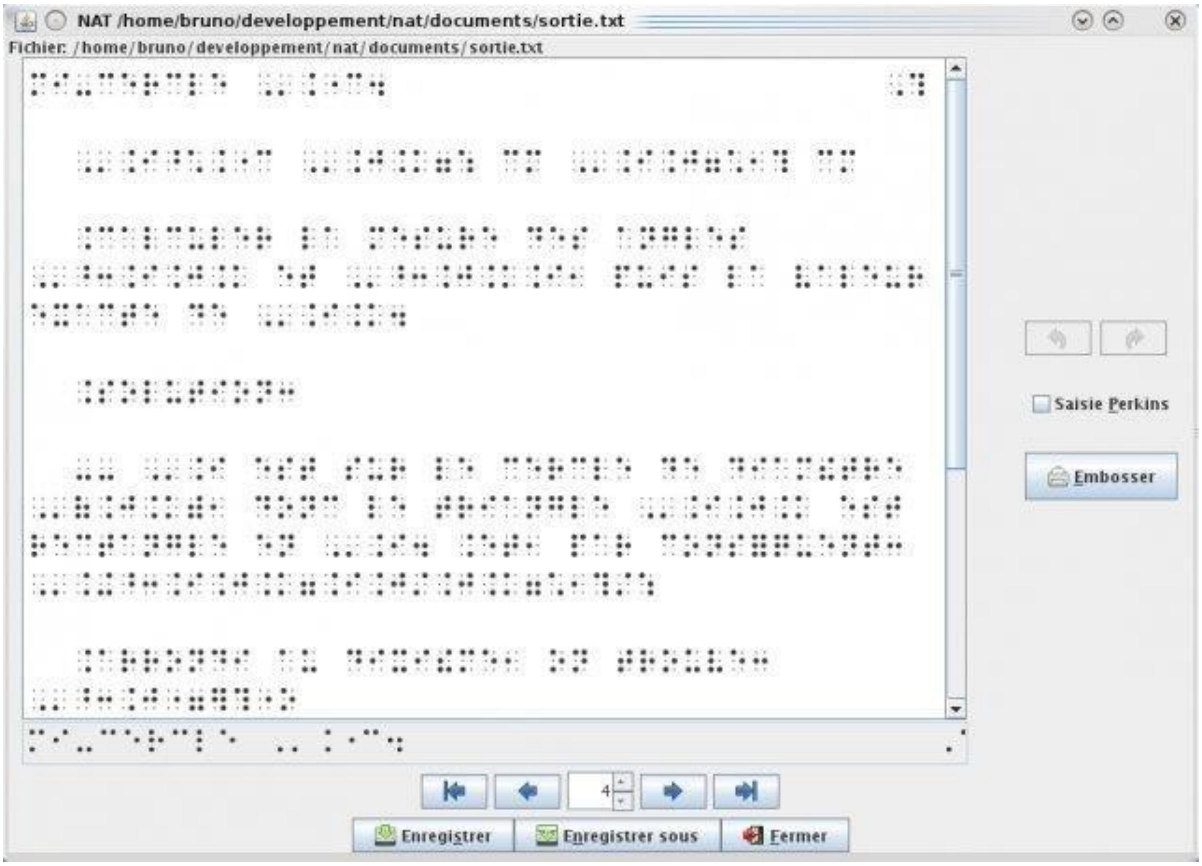


Figure 12: Interface NatBraille

NatBraille is free, open source transcription software for French Braille (and vice versa).

NatBraille was born in July 2008 on the initiative of the University of Lyon, it was funded by the Ministry of National Education and other entities.

NATBraille appears to have taken the place of the Bramanet (which is disappearing) by performing roughly the same functions.

In fact it is just a MathML to text converter according to the French Braile 6 point syntax.

It is a MathML to Braille text converter and therefore accepts all formats that include MathMl-based mathematical syntax, such as html, text formats, and MathMl as input. It also partially allows a conversion of elements of chemistry.

Ultimately what was said for Bramanet can also be considered for NatBraille

An association of NatBraille was born, whose goal is the promotion, development and training of NatBraille and any free software based Braille transcription application.

6.8 Maths Project - ICEVI

Development of a Mathematics Training Package for Teachers Working with Blind Students25

From the ICEVI website we find complete information on the Mathematics project which informs us that it is "*supported through the Higher Education Project by The Nippon Foundation, hopes to unlock the often hidden potentials of blind learners by building upon existing resources and developing a comprehensive package of training modules that will assist classroom teachers in learning how to create appropriate instructional adaptations that will make the learning of mathematical concepts easier and more enjoyable for blind learners, as well as their sighted classmates."*

ICEVI (International Council for Education of People with Visual Impairments), is an international association of individuals and organizations promoting the education of visually impaired children together with equal access to adequate education for all children and young people with visual impairments so that they can realize their full potential. ICEVI collaborates with international non-governmental development organizations and United Nations bodies (United Nations Economic and Social Council, UNESCO, UNICEF and WHO).

The Mathematics project starts from a joint project, (between ICEVI, Overbrook School for the Blind, and the Nippon Foundation), which in 2005 produced a guide entitled: Mathematics Made Easy for Children with Visual Impairment, which can be downloaded for free from the website of the ICEVI at this link:

http://icevi.org/wp-content/uploads/2019/05/Mathematics-Made-Easy-for-Children-with-Visual-Impairment.pdf

[25] https://icevi.org/maths-project/ (2021)

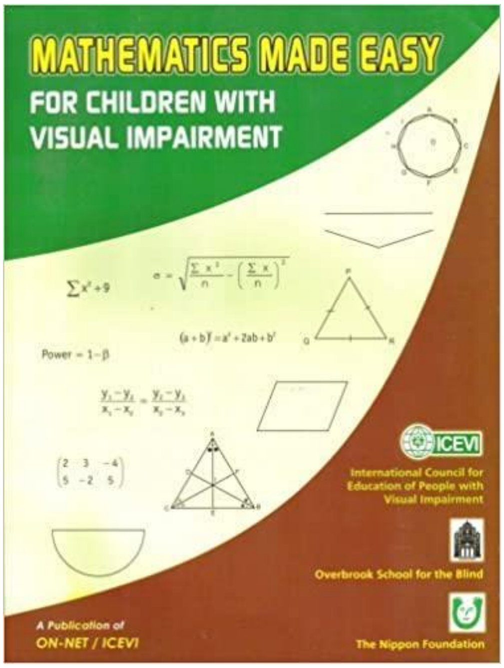


Figure 13: Cover of: Mathematics Made Easy for Children with Visual Impairment

Part of the 2005 project is also the creation of the **WIMATS** software to facilitate the printing on embossed Braille printers of the 6- dot mathematical Braille Nemeth. From a recent search on the web, the software does not seem to receive updates.

The guide is used in particular in Asia and has been translated into several other languages. It received criticisms and integration proposals, and in order to take these suggestions into account and to make the guide more usable (being able better to know tools and examples of work), the Mathematics project was born in 2016, with the aim of reviewing the topics in form of video lessons by creating a special you tube channel. Dr. MNG Mani is the Project Director, with the participation of ICEVI, Texas School for the Blind and Visually Impaired (TSBVI), Perkins School for the Blind and ON-NET and others.

The video-lessons are dedicated to mathematics teachers of blind children, but also of visually impaired children, starting from primary school upwards; they deal with topics of mathematics and geometry, using commercially available typhlological aids and materials, among which there is an intense use of 'abacus. From an analysis of the video guides, we notice that there are no computer aids or software dedicated to mathematics.

The YouTube channel link is the following:

https://www.youtube.com/channel/UCrmcpSzNg_9EXLbqExtVlAQ

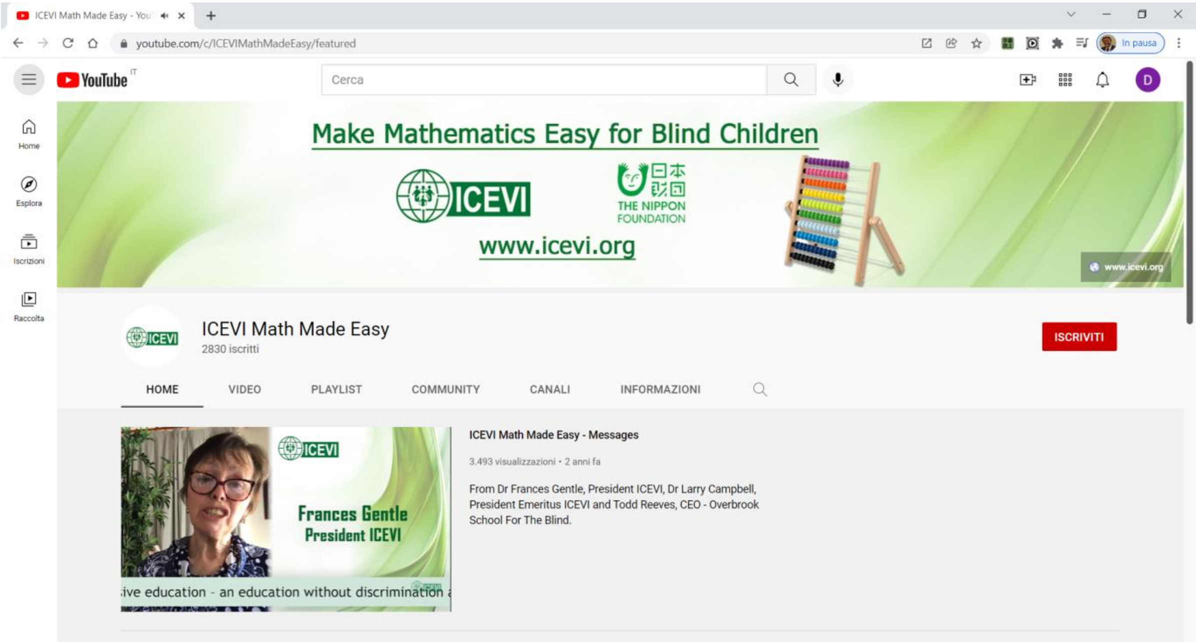


Figure 14: ICEVI Math project -Youtube channel

6.9 The Lambda System[26]

The European project LAMBDA has developed a system based on the functional integration of a linear mathematical code and an editor for visualization, writing and manipulation.

The symbols and special markers of LAMBDA have been defined thinking first of all of the 8-dot Braille combination that will represent them: all the operators, the most common markers and symbols are represented by a single character, chosen so as to be similar, as much as possible, to the corresponding 6-dot Braille character used in the various national codes in order to facilitate initial training and memorization.

[26] edited by Flavio Fogarolo

Even if the LAMBDA source code is unique (and the LAMBDA documents are therefore independent from national Braille choices), the Braille code applied to it changes from country to country to adapt, as best as possible, to national conventions and traditions.

For example, a compound fraction, with a numerator or denominator made up of several elements or expressions, needs three markers for the LAMBDA code: one indicating the beginning of the fraction, one indicating the fraction sign and therefore separating the numerator from the denominator, a third, finally, indicating the end of the fraction.

They will be managed by the program in a specific way and recorded on files with a code that identifies them uniquely but will appear on the Braille display in a different way depending on the country in which they are used. Furthermore, of course, the text descriptions and the voices associated with the speech synthesis will be read in the different languages.

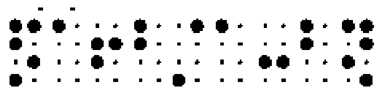
As regards, for example, the Italian version (the most widespread), the compound fraction uses three Braille symbols; for each of them we will also have a text in Italian for the name of the marker and one to indicate the words that the synthesis will have to pronounce in order to have a reading as continuous and natural as possible.

Braille	Braille Dots	Name of the marker	Text read by the speech synthesis
⡫	12467	Open compound fraction	Open fraction (or only "fraction" for the most experienced)
⡈	47	Intermediate compound fraction	Fraction sign
⢝	13458	Close compound fraction	End of the fraction

For example, the formula:

$$\frac{a+b}{a-b}$$

will appear in Braille, in the Italian 8-dot code:



As it is possible to see, the open / close pair has a strong analogy with the open and close numerator of the 6-dot Italian Braille (it was not possible to use the same symbols because the 8-dot code 1246 is assigned to the number 6).

The intermediate, i.e. the fraction sign, has a strong resemblance to the 6-dot bar (dots 34).

In other countries, different (even "very" different) rules should be followed to indicate the compound fraction in 6-dot Braille and therefore other combinations of Braille dots will be chosen to be assigned to the three LAMBDA markers.

Until now, the Lambda Braille code has been fully localized for the Italian one and partly for the Spanish one.

Since the available Braille combinations are fewer than the necessary symbols, it is inevitable, in some cases, to resort to combinations of Braille dots (two or more symbols in sequence to define a single element). In the definition of the 8-dot Braille code for mathematics, the number of Braille characters used are much fewer than the theoretically available 256 (28). Too many new symbols would have created too many problems of discrimination, memorization and training. The memorization of a completely new symbol is feasible for a limited number of cases; in general, when it is not possible to resort to analogies with the 6-dot code as seen above, it is better to try to exploit connections of a logical or mnemonic type. If, for example, we define a prefix to indicate the use of a symbol, as an analogy, in sets, we will be able to reuse a series of symbols which have already been known and memorized.

In this table we have some symbols defined in LAMBDA with the prefix set (dots 48).

$\cup$	Union	⢈⠖	Set theoretic prefix and addition
$/$	Difference between sets	⢈⠤	Set theoretic prefix and subtraction
$\subset$	Included strictly	⢈⣣	Set theoretic prefix and minor
$\subseteq$	Included in a broad sense	⢈⣣⠶	Set theoretic prefix and minor-equal

In addition to that for sets (dots 4 and 8 for Italian Braille), three other prefixes have been defined:

negation (dots 3468) reverses the meaning of the following symbol (for example: not equal, does not belong ...);

Greek (dots 45) to represent the Greek letters; it must be followed by the corresponding Latin letter, both capital and small;

generic (dots 34568) used in multiple contexts, especially geometry and logic.

Other double symbols are simple and intuitive, often already used in 6-dot Braille. Eg : $\leq$ (greater equal) will be <=, $\pm$(plus and minus) becomes +-, $\ll$ (much greater) will be << (two symbols of greater).

Interestingly, even if represented with more characters, symbols and markers are always considered as a unit (they must be inserted, deleted, moved, selected ... as if they were a single character). Also, the text-to-speech will always read the name of the element, not the sequence of symbols (for example, it will say *gamma* not *Greek-prefix g, union* not *set theoretic- prefix addition*, *much greater* not *greater greater*).

Even if LAMBDA is intended for the blind, its documents must also be usable by those who can see, through the screen, or by a normal ink printer.

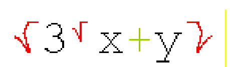
In the didactic field, fundamental is the contribution of the teacher, who must be able to follow the entire didactic process, not just examine and evaluate the final work.

What most characterizes the way of doing mathematics of blind students is the linearity of their code, not so much the use of Braille and other special equipment. To be truly helpful, the teacher must understand all the consequences that this type of approach entails; for example, the need to use markers that are not necessary in graphical notation, the risks of errors related to their use, the greater difficulties encountered in working with fractional objects (for example, identifying the common denominator to add algebraic fractions), the strategies to employ in order to manipulate the document using the keyboard instead of pen and paper. Students must be helped in this and it will obviously be up to their teacher to do it. To allow this, the LAMBDA system shows the mathematical text on the screen in linear mode, in full correspondence with what appears on the Braille display, using a textual graphic font. Symbols that do not have a conventional representation are represented with expressly designed characters that show the meaning of the text as clearly and immediately as possible.

For example, the formula:

$$\sqrt[3]{x+y}$$

will appear on the screen like this:



For those who can see, the linear version is undeniably less immediate than the other but, with a short training and a little effort, it is still easily accessible; this way, however, we have kept all the information on the way the pupil represents and manages the formula. Thanks to compatibility with MathML, with LAMBDA it is also possible to obtain traditional visualization in graphic mode, both on screen and on paper. A good quality document is obtained but unfortunately not accessible to the blind; it will only be used when it is really needed, for example to distribute the finished work in a wider context, outside the school environment.

The LAMBDA editor

The LAMBDA editor has a very similar organization to the one of the most common text management programs.

All the most frequent operations, such as opening a file, saving it, selecting a portion of text, deleting, correcting, copying, deleting, pasting ... are performed according to the standard Windows procedures and therefore do not present training or adaptation problems.

In the management of mathematical elements and, above all, of structures, the LAMBDA environment offers many more tools for writing, analysis and manipulation.

The LAMBDA editor, and this is its main feature, recognizes open / closed structures and provides various tools to manage in an easy and friendly way the variously interconnected and nested expression blocks they define.

In graphic mode, the structure of an expression is often immediately understandable, at least in its main characteristics.

Let's take this expression as an example:

$$\sqrt{\frac{(x+1)^2}{(x+1)(x-1)}+\frac{x^2}{x-1}}$$

Thanks to the visual approach, its global structure is immediately grasped: one understands on the fly that all the expression is placed under the root and that the rooting is made up of the sum of two compound fractions and a quick analysis of the content is enough to immediately identify some effective simplifications that quickly lead to a common denominator.

The situation with linear notation is completely different:

```
√⫽(x+1)^2∅(x+1)(x−1)⑊+⫽x^2∅x−1⑊⌉
```

By analyzing this formula in a sequential way (the only exploratory modality allowed by touch or by vocal reading) the structural information that was previously immediate and evident must now be reached in various steps: decode the symbols in succession one by one, mentally reconstruct the overall object, analyze this mental image to understand its structure and internal relationships. In order to know, for example, that the whole expression is under the root, it is necessary to complete the sequential analysis until you get to the last marker (close root) and mentally connect the two opening and closing markers and the text they embed.

The analysis of blocks is an operation that must often be repeated several times, at various levels in succession one inside the other, and it is precisely here that the main operational difficulty for blind students in mathematics very often lies.

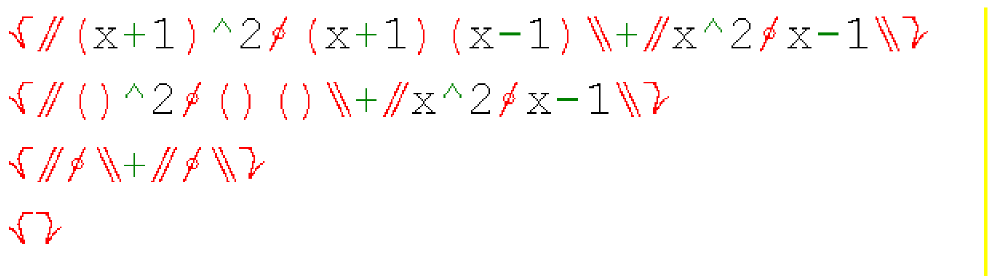
The management of blocks

An efficient management of the blocks of linear mathematical writing is one of the main objectives of the LAMBDA system.

The editor cannot be just a system for recording a sequence of characters, as for a normal word processor, but it must recognize blocks, that is, know which "close" marker each "open" is linked to, and vice versa, as well as to what eventual "intermediates". At this point you can create a whole series of support tools, for example commands to select (and so delete, copy, move ...) a whole block, to switch from a marker to the one linked to it, to delete all the markers of a block with a single operation (useful, for example, to simplify an expression without risking to leave unnecessary markers) ...

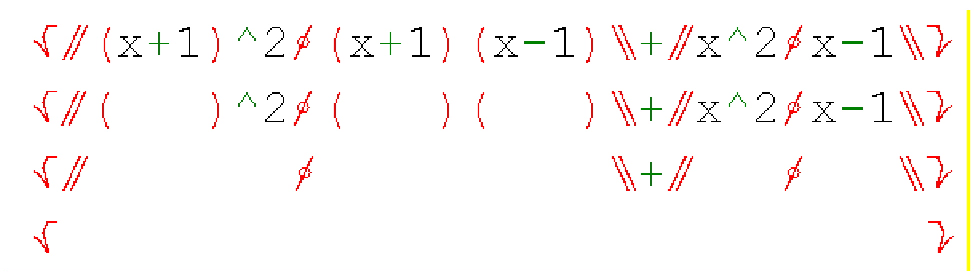
Of particular importance are the commands for alternative views, with which it is possible to hide the content of the blocks, by choosing the level of depth, to bring out the structure of an expression and facilitate understanding**.**

The following example shows the transformation of the formula already seen above, from the normal presentation (first line), to the first compression level (second line) up to the maximum level (last line).



Similar is the view that hides the contents of the blocks but preserves the empty spaces and is useful, if the formula is not too long, to get information also on the dimensions of the hidden blocks.

Here are the transformations of the same formula with this method (expanded structure):



In both cases, at the highest level these presentations show only the open-closed blocks of the square root and thus make us understand that this whole expression is under the root. Moving on to the second level, we can see that inside the root there is the sum of two compound fractions and so on.

In practice, an alternative way is offered to get to the understanding of the formula, a powerful resource in addition to the sequence that goes through the construction of the mental image.

The problem of errors

The problem of involuntary typing errors is very relevant for a blind user who uses the computer with only the Braille display, since the hands engaged in writing on the keyboard can check the entered text only at a later time. In a mathematical text, synthetic and without redundancies, these errors are very harmful and difficult to discover. A typing error in a literary text (if, for example, "problm" instead of "problem" is written) almost never changes the meaning of the sentence and is easy to identify, thanks to the context, even at a later time; a typo in mathematics (for example x + 1 instead of x-1) distorts the meaning of the expression and can only rarely be identified from the context.

Preventing these errors is therefore of fundamental importance.

A powerful control tool offered by LAMBDA is the echo in writing: the synthesis pronounces the name of the elements and markers that are inserted, whether they are typed through the keyboard, directly or through hotkeys, or chosen from the menu.

If you cannot or do not want to use the synthesis, it is also important to be able to type with the right hand only, through the numeric keypad, so that the left hand can control what appears on the display in real time. It takes some initial training but it's worth it.

There is another type of error that is related to the use of the computer, and not to mathematics, and therefore exclusively concerns the blind student, saving his classmates (one more reason to try to eliminate or at least reduce it). We are talking about respecting the syntax of the LAMBDA code, in particular of the block structures that must always be closed correctly, by inserting the appropriate marker. If the blocks are not correct (for example, a marker is missing or you put a closing marker that has nothing to do with the open structure) it will not be possible to apply the functions of LAMBDA that rely on blocks (for example the alternative views) and not even those that require conversion to MathML.

As usual, we have to keep in mind that the problem does not concern only the initial writing of the formula but also all subsequent elaborations and transformations. For example, if we want to simplify a compound fraction and leave only the numerator because the denominator has become equal to one, we must be sure to cancel all three markers, otherwise the overall structure of the formula remains unbalanced. These are operations that will be performed very often, in different situations, and cannot overlap with the real difficulties related to learning mathematics.

The problem is addressed by LAMBDA quite effectively:

• the insertion of the closing blocks and any intermediate blocks is assisted by the program; the user gives a generic closing (or separation) command and the program inserts, according to the context, the appropriate marker. Of course, if no block is open, nothing is inserted, and the error is signaled;

• if you try to leave a line without having closed all the markers, the command is blocked and the error is signaled;

• in the text processing phase, the markers can be eliminated only through the command that simultaneously deletes all the ones of a block, so that there are no isolated markers that would upset the structure.

This way, structural errors would in fact be impossible. However, there are some constraints that some more advanced and demanding users may refuse and for this reason all the functions related to the control of structural correctness are subject to the user's discretion and can be configured in the personalization menu.

We have examined so far the errors directly related to the use of the computer and the LAMBDA code, but dealing with mathematics at school, it is clear that we will also have to take into consideration the errors related to the discipline, from the ones of calculation to those of execution or correct application of rules and procedures.

The purpose of LAMBDA is certainly not to replace students but, also in this case, to provide an effective tool to allow them to do, in a different way but possibly with the same effort, what the others are doing.

It is essential to be able to review the work done and retrace the various steps. In this case, no special tools are needed but rather an effective strategy based on copy and paste work and subsequent transformations. To speed up these steps, a direct command is provided to copy and paste down, once or twice, the line on which the cursor is located.

However, these operations are closely related to personal preferences and LAMBDA provides the tools to deal with them with extreme flexibility, including the ability to record a series of commands in a macro and assign a combination of hotkeys to it.

7 Mathematics education and technologies for inclusion (VIP, SLD and other SEN)27

7.1 Competence in mathematics

In Italy, the data of the latest edition of 2018 of the OECD PISA (Program for International Student Assessment) international survey, aimed at detecting the skills of 15-year-old students in Reading, Mathematics and Science, highlighted that:

- ✓ Italian students obtained an average score in the PISA mathematics tests in line with the average of the OECD countries (Italy 487 vs OECD 489).
- ✓ From 2009 to date, the trend of PISA results in mathematics has remained constant.
- ✓ About 24% of 15-year-old Italian students have not reached Level 2, the basic level of proficiency in mathematics (OECD average 22%), while about 10% are in excellence levels 5 and 6 (OECD average 11%).

In general, it emerges that 1 out of 4 students does not reach the basic level of mathematical competence, both in Italy and in OECD countries.

In the 2012 edition, anxiety towards mathematics was also investigated by the OECD and it was found that:

- ✓ In OECD countries, 30% of students report feeling incapable when doing math problems; in Italy, the percentage of students in difficulty is 43%. Similarly, in Italy 43% of students report becoming very nervous when doing math problems (the OECD average is 31%).

This data highlights that in Italy students generally have less confidence in their ability to solve a set of pure and applied mathematics problems than the average of OECD countries.

The national guidelines for the nursery school and first cycle education curriculum (MIUR, 2012) lists the learning goals and objectives that students

[27] Shared activity with ESSEDIQUADRO – Dott.ssa Rebecca Tarello ITD-CNR

should achieve at the end of each cycle. The objectives of the first cycle (primary and lower secondary school) are divided into 4 areas: Numbers, Space and figures, Relations and functions, Data and forecasts.

At the end of the third grade of lower secondary school, therefore at the end of the first cycle of education, students should achieve, and then maintain and consolidate in subsequent years, the following objectives:

Numbers:

- ✓ Perform additions, subtractions, multiplications, divisions, sorts and comparisons between known numbers (natural numbers, whole numbers, fractions and decimal numbers), whenever possible in mind or using the usual written algorithms, calculators and spreadsheets and evaluating which tool may be more appropriate.
- ✓ Give rough estimates for the result of an operation and check the plausibility of a calculation.
- ✓ Represent the known numbers on the line.
- ✓ Use graduated scales in contexts relevant to science and technology.
- ✓ Use the concept of relationship between numbers or measures and express it both in decimal form and as a fraction.
- ✓ Use equivalent fractions and decimal numbers to denote the same rational number in different ways, being aware of the advantages and disadvantages of the different representations.
- ✓ Understand the meaning of percentage and know how to calculate it using different strategies.
- ✓ Interpret a percentage change in a given quantity as a multiplication by a decimal number.
- ✓ Find multiples and divisors of a natural number and multiples and divisors common to several numbers.

- ✓ Understand the meaning and usefulness of the smallest common multiple and the largest common divisor, in mathematics and in concrete situations.
- ✓ In simple cases, decompose natural numbers into prime factors and know the usefulness of this decomposition for different purposes.
- ✓ Use the usual notation for powers with positive integer exponents, aware of the meaning, and properties of powers to simplify calculations and notations.
- ✓ Know the square root as an inverse operator of the squaring.
- ✓ Give square root estimates using multiplication only.
- ✓ Know that you cannot find a fraction or decimal number that squared gives or other integers.
- ✓ Use associative and distributive properties to group and simplify operations, even mentally.
- ✓ Describe the sequence of operations that provide the solution to a problem with a numerical expression.
- ✓ Perform simple calculus expressions with known numbers, being aware of the meaning of parentheses and precedence conventions.
- ✓ Express measures also using powers of 10 and significant figures.

Space e figures:

- ✓ Reproduce geometric figures and drawings, using appropriate tools (ruler, square, compass, protractor, geometry software) in an appropriate way and with accuracy.
- ✓ Represent points, segments and figures on the Cartesian plane.
- ✓ Know definitions and properties (angles, symmetry axes, diagonals ...) of the main plane figures (triangles, quadrilaterals, regular polygons, circle).
- ✓ Describe complex figures and geometric constructions in order to communicate them to others.

- ✓ Reproduce geometric figures and designs based on a description and coding made by others.
- ✓ Recognize similar plane figures in various contexts and reproduce an assigned figure to scale.
- ✓ Know the Pythagorean Theorem and its applications in mathematics and in concrete situations.
- ✓ Determine the area of simple figures by breaking them down into elementary figures, such as triangles, or using the most common formulas.
- ✓ Estimate down and over the area of a figure also delimited by curved lines.
- ✓ Know the number π, and some ways to approximate it.
- ✓ Calculate the area of the circle and the length of the circumference, knowing the radius, and vice versa.
- ✓ Know and use the main geometric transformations and their invariants.
- ✓ Representing three-dimensional objects and figures in various ways through drawings on the plane.
- ✓ Visualize three-dimensional objects starting from two-dimensional representations.
- ✓ Calculate the area and volume of the most common solid figures and give estimates of everyday objects.
- ✓ Solve problems using the geometric properties of figures.

Relations e functions:

- ✓ Interpret, construct and transform formulas that contain letters to express relations and properties in a general form.
- ✓ Express the relationship of proportionality with an equality of fractions and vice versa.
- ✓ Use the Cartesian plane to represent relations and empirical functions or functions obtained from tables, and know in particular the functions of the

type y = ax, y = a / x, y = ax2, y = 2n and their graphs and connect the first two to the concept of proportionality.

- ✓ Explore and solve problems using first degree equations.

Data and forecasts:

- ✓ Representing sets of data, including using a spreadsheet.
- ✓ In significant situations, compare data in order to make decisions, using distributions of frequencies and relative frequencies.
- ✓ Choose and use average values (mode, median, arithmetic average) suitable for the type and characteristics of the data available.
- ✓ Know how to evaluate the variability of a set of data by determining, for example, the range of variation.
- ✓ In simple random situations, identify elementary events, assign them a probability, calculate the probability of some event, breaking it down into disjoint elementary events.
- ✓ Recognize pairs of complementary, incompatible, independent events.

7.2 Differences between difficulty and disorder

If in the presence of learning difficulties, good results can be obtained in a fairly short time through educational and rehabilitative interventions, this does not happen in case of learning disabilities, characterized by specificity, resistance to treatment and neurobiological basis.

Developmental dyscalculia is part of the Specific Learning Disorders (SLD), protected in Italy by Law 170/10 (New rules on specific learning disorders in schools), by a consequent decree (DM 5669 of 12 July 2011) and Guidelines attached; compensatory instruments and dispensatory measures are envisaged for specific disorders.

SLD, being part of the specific developmental disorders, are counted among the SEN (Special Educational Needs), which require specific didactic and educational attention.

In the first Consensus Conference (2007), two profiles of Dyscalculia are distinguished: "*profiles characterized by weakness in the cognitive structuring of the components of numerical cognition (i.e basal numerical intelligence: subitizing, mechanisms of quantification, comparison, seriation, mind calculation strategies) and others that involve executive procedures (reading, writing and placing numbers in columns) and calculation (retrieval of numerical facts and algorithms of written calculation). There is also general agreement on excluding the difficulties of solving mathematical problems from the diagnosis*".

The concept of discrepancy is fundamental for the diagnosis: the impairment of the specific ability must negatively deviate by at least 2 standard deviations ("typical" distance of each single observation from the average) from the expected normative values for the age or class attended and the intellectual level must fall within the limits of the norm, therefore not being less than 1 standard deviation with respect to the normative values.

In the second Consensus Conference (2011) the distinction of the two different profiles no longer appears but the invitation to deepen the "core deficit" remains, as well as to investigating general skills such as Working Memory, Semantic Memory and Visuospatial Skills. The difficulty in solving arithmetic problems is still excluded from the diagnosis.

The data collected by MIUR, through surveys in schools, show a sharp increase in certified pupils, from 0.7% in the 2010/2011 school year to 3.2% in the 2017-2018 school year.

Wanting to dwell on the various types of disorder, the first data collected by the Miur date back to the 2013/2014 school year and over a period of 4 years a significant growth rate emerges, in particular for dysgraphia and dyscalculia (dysgraphia + 163.4 %, dyscalculia + 160.5%, dysorthography + 149.3%, dyslexia + 88.7%).

From the detection of the 2017/2018 school year, out of a total of 8,582,920 pupils, the ones with SLD were 276,109, of which 88,645 with a diagnosis of dyscalculia (MIUR - Management of Information and Statistical Assets)

(https://www.miur.gov.it/-/scuola-pubblicati-i-dati-sugli-alunni-con-disturbi-specifici-dell-apprendimento)

Although the increase may be due to a greater awareness of the phenomenon, many others could be the contributing causes such as, for example, the criteria used to formulate the diagnosis.

To define dyscalculia it is possible to refer to the two most used international systems for the definition of disorders: the ICD-10 (International Classification of Diseases) of the WHO and the DSM-5 (Diagnostic System Manual) of the APA (American Psychological Association)

In the ICD-10 the specific disorder of arithmetic skills or dyscalculia is found within the specific developmental disorders of school skills (F.81) with the nosographic code F. 81.2 and must be used whether the difficulties are dependent on the sense of number or they concern the calculation. This code, in fact, does not differentiate between types of calculation disturbances. It includes "*developmental arithmetic disorder, Gerstmann syndrome, developmental acalculia*" and excludes "*arithmetic difficulties associated with another reading or spelling disorder".*

In the DSM-5 the specific learning disorder with impaired calculus or dyscalculia is found among the learning disorders (315.00) with the nosographic code 3015.01 and includes, in addition to the difficulties in the concept of number, memorization of arithmetic facts, accurate or fluent calculation, even difficulties in correct mathematical reasoning.

In cases where the learning disorder is linked to below average cognitive profiles or to sensory causes (e.g. deafness, low vision), neurological (e.g.

epilepsy), genetic (e.g. Down syndrome, Williams syndrome), organic (e.g. hypothyroidism), psychological (e.g. primary psychopathological disorders), we speak of Aspecific (or non-specific) Learning Disorder and we do not refer to the legislation on SLD but, in case of disability, to law 104/92, or in other cases, the directive relating to the other SEN of 27 December 2012 and CM of 6 March 2013.

It therefore emerges that the calculation disorder has heterogeneous clinical manifestations, which are due to the complexity of the underlying mechanisms. The skills in mathematics are, in fact, only partially domain-specific (number area) since they are based on other skills of a transversal character such as linguistic, visual-spatial, attentional, mnemonic, praxic, logical and executive skills.

7.3 Cognitive and neuropsychological models

The interpretative models of evolutionary dyscalculia have changed over time, following the flourishing of studies aimed at understanding the processes underlying typical and atypical development.

These models differ from each other in particular with respect to the importance given to the role of specific domain or general domain mechanisms in this process. (Girelli, 2013).

The first current, the so-called "innatist", believes that dyscalculia derives from a dysfunction in the basic mechanisms of quantification (Deahene 1997, Butterworth, 1999).

The second current, in contrast, tends towards a multifactorial etiopathogenesis that accounts for the complexity of the numerical abilities and clinical heterogeneity of the disorder (Szucks and Goswami, 2013).

The hypothesis that dyscalculia may derive from a primary deficit in basic skills, or "core systems", has been supported by studies on the ontogenesis of numerical skills which have led to the affirmation that there are two systems of representation of quantities and that they are at the basis of the triggering of the development of numerical skills: a system of identification of multiple objects or OTS (Object Tracking System), which provides an exact representation of small quantities, and an approximate number system ANS (Approximate Number System), which provides an approximate representation of a set, whose functionality is expressed in terms of "numerical acuity" (Piazza et al, 2010).

From these premises, a line of research was born, aimed at verifying the correctness of the hypothesis of pure or primary dyscalculia, looking for evidence that it is associated with: neuro-anatomical indices of dysfunctionality or structural anomalies at the level of the intraparietal sulcus; behavioral indices of difficulty in basic numerical tasks (Biancardi et. al, 2017).

The models underlying the most accredited numerical processing and computation skills are:

- ✓ The model of McCloskey, Caramazza and Basili (1985)
- ✓ Dehaene's Triple Code Model (1992)

The modular model of the number processing system by McCloskey, Caramazza and Basili (1985) draws a fundamental distinction between the number processing system and the calculation system.

The number processing system includes a component for understanding and one for producing numbers, while the calculation system is made up of the facts and procedures specifically needed to perform the calculations (McCloskey, Caramazza, & Basili, 1985).

In the system responsible for processing numbers, both for understanding and for production, there are two subsystems: the subsystem of Arabic numbers and the one of verbal numbers, and two mechanisms are postulated for each: the lexical mechanism and the syntactic mechanism.

The systems for understanding and producing numbers provide the input and output to the calculation system which in turn includes functionally autonomous subcomponents, assigned to the recognition of the signs of the operation, the recovery of arithmetic facts and the procedures of calculation (Girelli, 2013).

Although the model was born to account for acquired disorders in adults, it was then applied for the interpretation of neurodevelopmental disorders by various authors including Temple (1989, 1991).

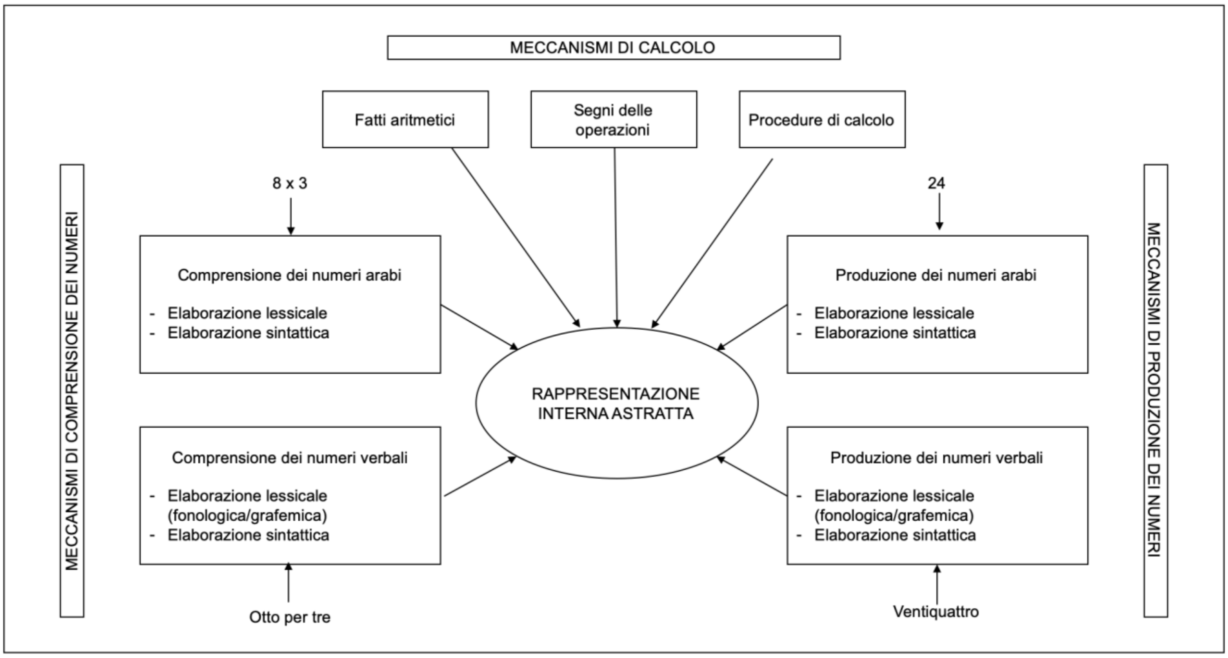


Figure 15: Reworking of the figure taken from Cornoldi 2007 page 105

The Triple Code Model for Numerical Cognition is based on two premises: that numbers can be presented mentally in three different codes (visual-arabic, auditory-verbal, analog of magnitude on a hypothetical mental "number line") and that each numerical procedure is linked to a specific input and output code (Dehaene, 1992).

Within the triple code model, each code is assigned to specific numerical tasks:

- ✓ skills such as the estimation of magnitude and the comparison of numbers (semantic information) are attributed to the analog module;

- ✓ skills such as counting and retrieving arithmetic facts are attributed to the auditory-verbal module;
- ✓ skills in solving written calculations or in judging the parity of a number are based on the visual-arabic module.

These three modules constitute a number processing and calculation system in which the modules are autonomous, interconnected and activated according to the particular needs of a given task (Von Aster, 2000).

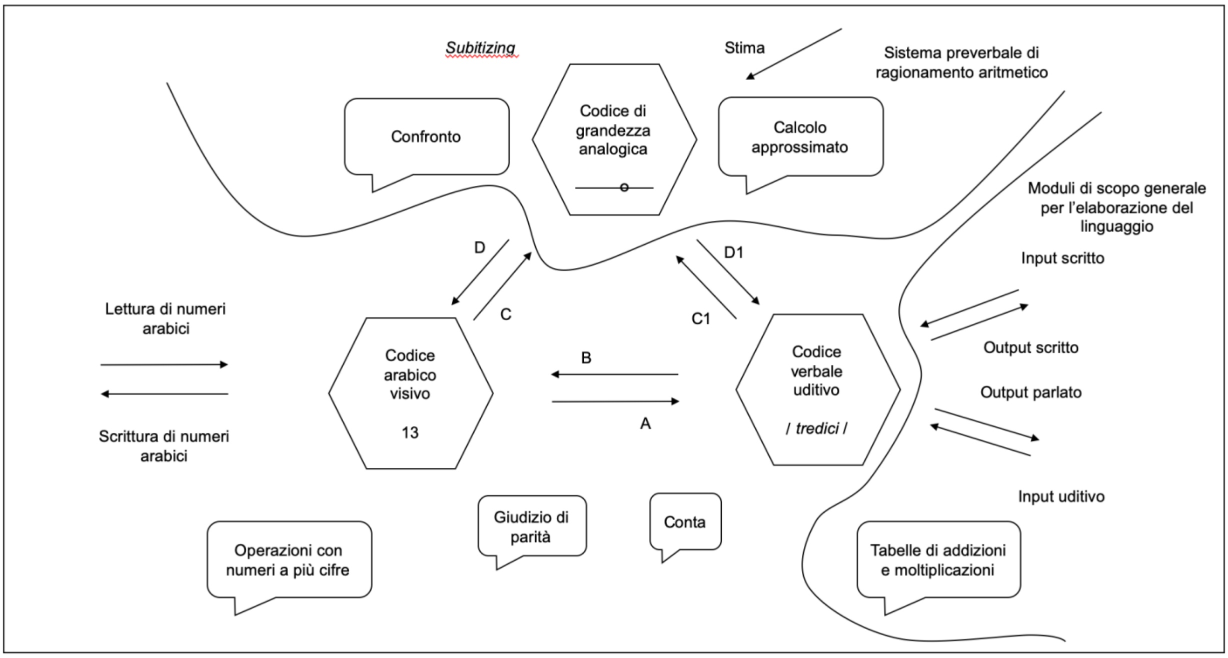


Figure 16: Reworking of the figure taken from Cornoldi 2007 page 107

Parallel to the research aimed at identifying possible highly selective functional deficits at the origin of developmental dyscalculia, a second line of studies has focused on the impact of other cognitive abilities such as working memory (Geary et al., 2002), linguistic competence (Carey and Spelke, 1996), and visuospatial skills (Rourke, 1993) in order to classify learning difficulties in mathematics (mathematical disabilities).

Given the great heterogeneity of the disorder, this line of research could be very promising.

8 Cataloging the available resources. Tools and solutions.

To catalog the available resources, we propose two taxonomies: one relating to compensatory software divided by functions and one relating to enabling software divided according to the areas to be strengthened.

In addition, tools accessible to the blind and visually impaired and a selection of additional tools for mathematics with coding and educational robotics have been included.

Compensatory instruments:

- ✓ Calculators
- ✓ Solvers of calculus and geometry problems
- ✓ Virtual manipulative
- ✓ Editor for mathematical notation
- ✓ Text-to-speech for mathematical notation

Enablement / enhancement tools:

- ✓ Number system
- ✓ System of calculation
- ✓ Space and figures

8.1 COMPENSATORY SOFTWARE

CALCULATORS				
QR SD2	**TITLE**	**DESCRIPTION**	**LEVEL**	**FOCUS**
	Desmos	Web app/App: collection of calculators and accessible tools	Lower secondary Upper secondary University	SLD Visually impaired Blind Other SEN
	Jumbo Calculator	App: essential calculator with large keys. It can export the results outside the app and supports speech synthesis	Lower secondary Upper secondary University	SLD Visually impaired Blind Other SEN
	Talking calculator	App: basic talking calculator with large, colored keys. It offers the possibility to adjust the black on yellow color contrast in addition to the speech synthesis. It has natural pronunciation and is compatible with Braille devices	Primary Lower secondary Upper secondary University	SLD Visually impaired Blind Other SEN

	Scientific Talking Calculator	App: scientific talking calculator with large, colored keys. It offers the possibility to adjust the black on yellow color contrast in addition to the speech synthesis. It has natural pronunciation and is compatible with Braille devices	Upper secondary University	SLD Visually impaired Blind Other SEN
	Statistic Talking Calculator	App: statistic talking calculator with large, colored keys. It offers the possibility to adjust the black on yellow color contrast in addition to the speech synthesis. It has natural pronunciation and is compatible with Braille devices	Upper secondary University	SLD Visually impaired Blind Other SEN
	GeoGebra Suite Calculators	WebApp/App: suite of calculators enabling to create graphs, solve equations, determine derivatives and integrals, make statistical calculations and build 2D and 3D geometric figures	Primary	SLD Other SEN

EDITOR FOR MATHEMATICAL WRITING				
QR SD2	**TITLE**	**DESCRIPTION**	**LEVEL**	**FOCUS**
	AAC - Accessible Equation Editor	Web App: mathematical expression editor based on the Nemeth Braille code	Lower secondary Upper secondary University	Visually impaired Blind
	Edico – Scientific Editor ONCE	Software: bidirectional math expression editor that transcribes standard text into Braille and vice versa	Primary Lower secondary Upper secondary University	Visually impaired Blind
	L-Math	Software: editor that allows the writing and reading of mathematical formulas through the BlindMath and TalkingMath modules. It also enables to explore 2D graphics with the BlindGraf (speech synthesis) and AudioTac (haptic feedback) modules	Lower secondary Upper secondary University	Visually impaired Blind
	Lambda	Software: text editor and mathematical notation in linear form, similar to MathML code, based on	Lower secondary Upper secondary	Visually impaired Blind

		an 8-dot Braille representation with 256 unique characters. It is integrated with speech synthesis.	University	
	Mod Math	App: math notation editor on a checkered grid. It is possible to transcribe both basic calculations and complex algebraic equations with a very intuitive system and export them to other programs	Primary Lower secondary Upper secondary University	SLD Visually impaired Other SEN

VIRTUAL MANIPULATIVES				
QR SD2	**TITLE**	**DESCRIPTION**	**LEVEL**	**FOCUS**
	Didax	Web app: collection of virtual manipulators for tablets or Miw	Primary Lower secondary	SLD Visually impaired Other SEN
	Math Learning Center	Web app/App: collection of virtual manipulators	Primary Lower secondary	SLD Visually impaired Other SEN

	Mathigon	Web app/App: interactive learning environment with games and a collection of Polypad virtual manipulators	Primary Lower secondary	SLD Visually impaired Other SEN

CALCULATION AND GEOMETRY PROBLEM SOLVERS				
QR SD2	**TITLE**	**DESCRIPTION**	**LEVEL**	**FOCUS**
	Microsoft Math Solver	Web app/app: step-by-step solving of math problems. It enables to view the steps and graphs and to learn more about the topic through linked lessons. It is integrated with Immersive Reader which allows accessibility	Primary Lower secondary Upper secondary University	SLD Visually impaired Other SEN
	PhotoMath	App: step-by-step solving of math problems. It is possible to manually enter the data in the software or acquire it with the camera from the app. It includes many functions ranging from	Primary Lower secondary Upper secondary University	SLD Visually impaired Other SEN

		simple to advanced		
	RisolviGeometria	Web app/App: step-by-step resolution of geometry problems. It is possible to draw on the figure to indicate the data, rotate and resize it.	Primary Lower secondary Upper secondary	SLD Visually impaired Other SEN

VOICE SYNTHESIS FOR MATHEMATICAL NOTATION				
QR SD2	**TITLE**	**DESCRIPTION**	**LEVEL**	**FOCUS**
	AsTeR	Software: speech synthesis for textual and mathematical documents written in TEX languages (e.g. Latex)	Lower secondary Upper secondary University	Visually impaired Blind
	AudioMath	Software: speech synthesis for mathematical documents written in MathML language	Lower secondary Upper secondary University	Visually impaired Blind
	Matvox	Software: speech synthesis for mathematical documents written with the Edivox editor. Both are part of the Dosvox suite	Lower secondary Upper secondary University	Visually impaired Blind

	ChattyInfty	Software: speech synthesis for mathematical documents scanned with InftyReader or edited in Infty Editor using the BlindMath module in Latex. It also enables to explore 2D graphs via BlindGraph	Lower secondary Upper secondary University	Visually impaired Blind
	Math Talk	Software: Speech synthesis for mathematical documents. It enables to translate mathematical notations into Braille	Lower secondary Upper secondary University	Visually impaired Blind

8.2 ENABLING / ENHANCEMENT SOFTWARE

NUMBER SYSTEM				
QR SD2	**TITLE**	**DESCRIPTION**	**LEVEL**	**FOCUS**
	1 to 100 Numbers Challenge	App: enhancement of number recognition and counting up from 0 to 100. The exercise consists in selecting the numbers randomly arranged on a matrix in ascending order	Primary Lower secondary Upper secondary University	SLD Other SEN
	Subitize This!	App: strengthening of the sense of number with subitizing exercises from 1 to 5 elements presented in patterns of 4, 6 or 8. Feedback is provided on the answers and the time taken	Nursery school Primary	SLD Other SEN
	Pattern Sets	App: strengthening of the sense of number with exercises to recognize the number of points (from 1 to 10) in	Nursery school Primary	SLD Other SEN

		different patterns. The time of presentation of the stimulus can be varied		
	Montessori Math	App: innovative approach to the decimal system and the positional value to count up to 1000. The materials are those of the Montessori methodology: golden pearls and Seguin table. As the game levels are unlocked, an interactive city is built	Primary	SLD Visually impaired Other SEN
	Dexteria Dots	App: math game to train the sense of number and learn the concepts of addition, subtraction and relative dimension while also exercising visual perception and fine motor skills	Nursery school Primary	SLD Visually impaired Other SEN
	Dexteria Dots 2	App: math game to learn the concepts of greater, less and equal while also exercising visual perception and fine motor skills	Primary	SLD Visually impaired Other SEN
	Quick Math Jr. School Edition	App: exercises on the semantic coding of	Nursery school	SLD Visually

		numbers	Primary	impaired Other SEN
	Number Line Math 3-6	App: exercises of arranging numbers in the correct sequence and counting on the number line. It is possible to include negative numbers as well	Nursery school Primary	SLD Visually impaired Other SEN

CALCULATION SYSTEM				
QR SD2	**TITLE**	**DESCRIPTION**	**LEVEL**	**FOCUS**
	Animal Watch VI Suite	App: mathematical problems with the common thread of endangered animals	Primary Lower secondary	Visually impaired Blind
	Hand-On Equations 1 e 2	App: increasingly complex equations to be solved through virtual manipulators. Based on Dr. Borenson's tangible program	Primary Lower secondary	SLD Visually impaired
	Math Melodies	App: mathematics exercises within the	Primary	Visually impaired

		narrative framework of some stories that can be read or listened to with the vocal synthesis		Blind
	iMatematica	App: exercises on various topics, form divided into sections, utility to help with calculations	Primary Lower secondary Upper secondary University	SLD Other SEN
	Gimme Five	App: exercises to support the development of strategies for the mental calculation of addition and subtraction by 10, 100 and 1000	Primary	SLD Visually impaired Other SEN
	In volo con la matematica	App: mental arithmetic exercises based on the principles of the analogical method of dr. Bortolato and on the Line of 20	Nursery school Primary	SLD Visually impaired Other SEN
	Kids Maths – App educative	App: quiz with the 4 arithmetic operations with a very basic interface and large, coloured buttons	Primary	SLD Other SEN
	MathBoard	App: exercises with column operations to build and consolidate the procedures of the	Primary	SLD Other SEN

		calculation written on a virtual slate blackboard		
	Fractions & Shapes	App: set of 3 digital notebooks with exercises to understand the meaning of denominator and numerator and calculate sums and subtractions between fractions	Primary	SLD Other SEN
	Multiplication Tables & Apples	App: set of 10 booklets with 90 total exercises with increasing difficulty to learn the times tables from 1 to 10. You start by counting apples in a bucket	Primary	SLD Other SEN
	Decimals & Fractions	App: set of 32 lessons with instructions and 5 tests to understand and practice calculating with decimal numbers	Primary	SLD Other SEN
	Math Chase	Web app: collection of about 100 calculation exercises in mind. It enables to practice by varying the maximum response times with exercises of increasing difficulty and to save the data after registration.	Primary	SLD Other SEN

SPACE AND FIGURES				
QR SD2	**TITLE**	**DESCRIPTION**	**LEVEL**	**FOCUS**
	Geometria Facile 1	Software: program of exercises on basic geometric concepts that can also be done in animated story mode. It is accompanied by a management system	Primary	SLD Visually impaired Other SEN
	WeDraw	WebApp: game suite for multisensory learning of arithmetic and geometric concepts developed by the IIT	Primary	SLD Visually impaired Other SEN
	Montessori Geometry	App: games to learn 23 shapes belonging to 6 different families and recognize them inside images with well-finished graphics	Nursery school Primary	SLD Visually impaired Other SEN

8.1 SOFTWARE AND HARDWARE FOR CODING AND ROBOTICS FOR VIP (VISUAL IMPAIRED PEOPLE)

VISUAL + AUDITORY				
QR SD2	**TITLE**	**DESCRIPTION**	**LEVEL**	**FOCUS**
	Swift Playground	Software: textual programming environment that supports speech synthesis	Nursery school Primary Lower secondary Upper secondary	Visually impaired Blind SLD Other SEN
	Scratch 3	Software: visual block programming environment with speech synthesis function	Nursery school Primary Lower secondary Upper secondary	Visually impaired Blind SLD Other SEN

VISUAL + TANGIBLE				
QR SD2	**TITLE**	**DESCRIPTION**	**LEVEL**	**FOCUS**
	Code Jumper VI	Software e Hardware: programming environment with tangible blocks with explanations also translated into Braille	Nursery school Primary Lower secondary Upper secondary	Visually impaired Blind SLD Other SEN
	Makey Makey	Hardware: card that enables to transform an object made of any conductive material into a controller. It can be connected as an extension to Scratch3	Nursery school Primary Lower secondary Upper secondary	Visually impaired Blind SLD Other SEN

9 Conclusions

The analysis of the state of the art on the subject of mathematics education and accessibility for blind students, has taken into consideration many aspects of the issue starting from the codes, from existing software, to research carried out in university laboratories and other national and European projects. .

The analysis showed that in the 1990s and 2000s there was intense research that led to the design and testing of various prototypes based on the 6-dot and then 8-dot code, readers of LaTeX. However, today many of those researches have remained at the level of thesis and of doctoral thesis, of prototypes, without becoming really usable products with a guarantee of assistance, maintenance and updating over the years. In the following years, further research initiatives have reduced the scope of work on a more limited number of proposals which can be summarized as follows:

1) In the age group corresponding to the years of primary school attendance and up to the beginning of upper secondary school, national 6-dot Braille is adopted, using mostly embossed paper, Braille-typing, other instruments such as the cubarithm or the dactyl-rhythm (if still available as the manufacturer closed the production several years ago).

2) In high school, LaTeX is introduced gradually, in particular for more capable children, especially in Germany, Austria, Poland.

3) in France a computer system based on the 6-dot national coding is widespread.

4) in Italy the Lambda system is used by over 90% of the students of lower and upper secondary school, but it also finds users in Spain, Germany and the Czech Republic.

Mathematical notation (meaning terminologies and procedures) with a few exceptions such as, for example, the period used in Anglo-Saxon countries instead of the comma, and vice versa, is universal. This universality does not correspond to the mathematical Braille writing, as each country adopts a mode of representation using the 63 national Braille signs and a structural organization adapted to the language and tradition initiated by national libraries or by national blind associations.

As a result, there is no compatibility between mathematical texts in Braille produced in countries with different languages. Furthermore, attempts to devise an 8-dot Braille universal syntax and encoding were unsuccessful and remained a purely academic exercise. Likewise, there is no electronic coding in XML of Braille mathematics (at least for the linear form and structure, since the signs can be modified thanks to tables) with the exception of XML Lambda coding.

A similar standardized Braille representation exists, for example, for Braille music. This happened in 1996 thanks to the tireless work of the musician Bettye Krolig who devoted years of her professional activity to this laudable goal, namely that of being able to get the approval of a common musical Braille code, recognized in the main countries of the world, to be exact 16 countries. This is the New International Manual of Braille Music Notation[28] which was approved by a working group on Braille music, sponsored by the World Union of the Blind, the WBU. Furthermore, after more than thirty years, this manual is undergoing a series of amendments and corrections by the Braille Authority of North America, which will propose them to an upcoming assembly of the WBU music. This example makes it clear that the result of the standardization of a Braille encoding was made possible only thanks to the patient work of an expert musician, who created a working group which was to become a commission of the WBU. The problem of musical Braille dialects and of the incompatibility between transcriptions has finally been solved, even if the manual presents many compromises that do not make it so homogeneous and uniform, but it is a path studded with compromises, which will gradually be solved. Not only that, having a musical Braille standard has encouraged computer research in this sector, in order to design Braille music writing programs such as BM2021 (Braille Music 2021), which was born as a result of two European research projects; it is usable in all languages, as the writing of

[28] http://www.bibliotecaciechi.it/new-international-manual-braille-music-notation (2021)

music is a standardized graphic representation. The comparison makes it clear that, within the WBU, it would be desirable to create a working group, able to deal with shared Braille mathematical coding without language barriers, thanks to new technologies.

The different European countries, on the other hand, up to now have maintained local mathematical solutions and Braille codes, therefore a process has been lacking that could have led to an efficient unitary system at a general level capable of promoting the exchange of texts and a shared teaching methodology.

Blind children who study music are a fraction of the totality that has to face the study of mathematics throughout the course of school. So the actors are much more numerous and besides the students and their families, the teachers of schools (curricular, support, study assistants), the staff of libraries and transcription centers, the managers of the user associations, come into play and certainly a connection activity between all these actors is much more complex than within the restricted world of blind musicians alone.

The DDMATH project has collected the most used researches and products, the needs of teachers, students and families, and contributes in this awareness-raising process for a European debate that is able to direct research towards new proposals and efficient solutions in order to guarantee wide access to mathematical documents to blind students, perhaps with a new mathematical notation.

Ultimately, the cardinal points of the problem are two:

1) as concerns schools, in our opinion, they are the **availability of an efficient mathematical editor** and a unitary mathematical **Braille code**.

2) With regard to universities and scientific courses, the solution that seems to be most adopted and preferred is the use of LaTeX.

If we limit our analysis to the first point, by **code** we mean the set of rules that must be followed to represent a mathematical text in linear format; closely related to the code is the problem of transferability, i.e. the possibility of importing and exporting documents in different formats. To be fully

transferable, the code must be **complete**, i.e. it must include all the mathematical elements.

If it has 8 dots, a further problem is added, namely the one of transforming in **Braille printing** according to the chosen 6-dot national code.

There are different approaches to the 6-dot press issue. In some countries, such as in Italy, the use of an efficient mathematical editor has in fact made the need for printing on 6-dot embossed paper superfluous, preferring by far only the electronic format, as it is happening for all alphanumeric texts. In Portugal, as well as in France, the printing of 6-dot mathematical texts (exercises, theory) is still widely offered to students.

In those countries where LaTeX is adopted, especially for university studies, there are problems related to complex conversions and transformations into different formats including 6-dot Braille, which are not always possible since LaTeX is a display code, and therefore also in these cases it is preferred the use of the LaTeX electronic format only.

Still considering the possible adoption of an 8-dot code, there is also the problem of the way this code can be displayed in **8-dot Braille on the computer, because in each country a correspondence is adopted between the alphanumeric character and the Braille sign, depending on the language of the country.**

Therefore, the availability of an efficient **editor** capable of managing all these aspects related to the language becomes a fundamental issue for the solution of the problem.

Theoretically, linear math text is able to render any mathematical expression but in reality there are tough efficiency problems to overcome, in particular in relation to manipulating the maths expression (i.e. the series of operations that must be carried out to solve the expression or equation, demonstrate a theorem and so on) and in the understanding of complex structures and relations, above all those related to visual graphic representation.

It follows that the code itself, even if innovative, if at 8 dot and if common to all countries, alone could not have any advantage if not coupled with an efficient Braille mathematical editor.

The 2006 European Lambda project was the first to understand that the heart of the problem was linked to the availability of a mathematical Braille editor. In fact today, after more than twenty years, this intuition is confirmed by the fact that well-structured 8-dot codes (such as the SMSB code for example), in the absence of an editor capable of managing them, have not found practical use among users and schools. .

The Lambda editor meets the needs of users and teachers, especially Italian, as it allows for:

- efficient and controllable insertion of **symbols and markers not present on the keyboard;**

- presence of **manipulation** tools;

- presence of tools for the **analysis and understanding of an expression**.

Furthermore, as insistently requested by teachers in Italian integrated schools, **the Lambda editor and mathematical code guarantee full transparency of the mathematical text to a sighted person** who is consulting or managing the program via a monitor and has no knowledge of Braille. It is not just a matter of understanding what is being shown, but also of actively intervening in the didactic process.

In the future, it is desirable the research to continue with further innovative initiatives along the way started by the Lambda project, which has proposed a solution for students of very different ages and skills, from middle school to university. It will be essential to be able to adapt the environment to the individual pupil, providing the tools that are really needed in that phase of study and offering the possibility of applying different strategies to achieve similar results.

Research in this area requires new energies and resources, as the points to be analyzed are really many, given that the normal technical and usability aspects of a computer application are supplemented by important didactic evaluations and a series of considerations related to the consistency of the systems with the Braille rules adopted in the different countries.

It is clear that the results of didactic considerations, linked to a learning process, can be evaluated after a few years and certainly not in few months. Furthermore, good technical solutions that do not claim to change the way of teaching mathematics to the blind are not enough, even if systems of this type have important repercussions on teaching too, as indeed the computer itself had in other subjects. It is important to manage and not suffer these relapses, avoiding technicalities and always starting from the point of view of the pupil. Some unsolved problems still remain, that could in the future find a solution, or at least some help, from computer science: for example, the management of graphs in the study of functions and the access to mathematics by the visually impaired. A serious collaboration between people involved in technologies for the blind and the world of school appears increasingly essential in order to ensure that the potentials of the new tools become determining factors for growth and for the development of autonomy

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